3.5 Limits at Infinity

$\lim_{x\to\infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x)=\frac{5x^2-3x+4}{2x^2+5x}=\frac{5-\frac{3}{x}+\frac{4}{x^2}}{2+\frac{5}{x}}$$

$\frac{3}{x}\to 0$, $\frac{4}{x^2}\to 0$, $\frac{5}{x}\to 0$

horizontal asymptote: $y=\frac{5}{2}$

$\Rightarrow \lim_{x\to\infty} f(x)=\lim_{x\to-\infty} f(x)=\frac{5}{2}$

$$f(x)=\frac{2x-4}{3x^4}=\frac{2x/x^4-4/x^4}{3x^4/x^4}=\frac{\frac{2}{x^3}-\frac{4}{x^4}}{3}$$

H.A. $y=0$

$\lim_{x\to\infty} f(x)=\lim_{x\to-\infty} f(x)=0$

$$f(x) = \frac{2x^7 - 4x^3 - 2}{5x^4 + 1} \approx \frac{2x^7}{5x^4} \approx \frac{2}{5}x^3$$

$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

$$f(x) = \frac{2 - 7x^3 + 2x}{1 + x} \approx \frac{-7x^3}{x} = -7x^2$$

$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

24. $$\lim_{x\to-\infty}\left(\frac{1}{2}x - \underbrace{\frac{4}{x^2}}_{0}\right) = -\infty$$

26. $$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2}} = \lim_{x\to-\infty} \frac{x}{|x|}$$

$$= \lim_{x\to-\infty} \frac{x}{-x} = \lim_{x\to-\infty} (-1)$$

$$= \boxed{-1}$$

Recall:

$$\sqrt[n]{x^n} = \begin{cases} x & \text{if } n \text{ is odd} \\ |x| & \text{if } n \text{ is even} \end{cases}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\sqrt[3]{2^3} = 2 \qquad \sqrt[3]{(-2)^3} = -2$$

$$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2|$$

$$\lim_{x\to\infty} \frac{5x-2}{\sqrt{9x^2+3}} = \lim_{x\to\infty} \frac{5x}{\sqrt{9x^2}}$$

$$= \lim_{x\to\infty} \frac{5x}{|3x|} = \lim_{x\to\infty} \frac{5x}{3x} = \boxed{\frac{5}{3}}$$

30. $\displaystyle\lim_{x\to\infty} \frac{x-\cos x}{x}$

$\displaystyle= \lim_{x\to\infty} \frac{x}{x} - \lim_{x\to\infty} \frac{\cos x}{x} = \frac{[-1,1]}{\infty} \to 0$

$= 1 - 0 = \boxed{1}$

32. $\displaystyle\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)$

$\displaystyle= \cos\left[\lim_{x\to\infty} \frac{1}{x}\right] = \cos 0 = \boxed{1}$

18. c.

$$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}} =$$

$$= \lim_{x\to\infty} \frac{5x^{2/2}}{4} = \boxed{\infty}$$

$$= \lim_{x\to\infty} \frac{5x}{4}$$

note $\lim_{x\to-\infty} \frac{5x^{3/2}}{4\sqrt{x}+1}$ is undefined!

3.7 Optimization Problems

4. Find two positive numbers (x, y) whose product is 192 and the sum (S) of the first plus three times the second is a minimum.

$xy = 192 \qquad x = \frac{192}{y}$

$\frac{1}{y} = y^{-1}$

$S(x,y) = x + 3y$

$S(y) = \frac{192}{y} + 3y$

$S'(y) = -\frac{192}{y^2} + 3$

$-\frac{192}{y^2} + 3 = 0$

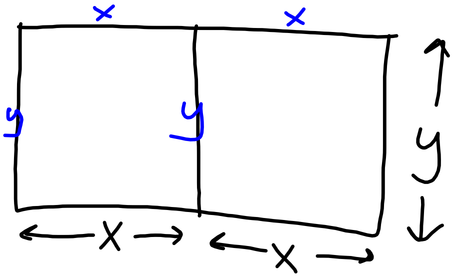
$-192 = -3y^2$

$\frac{192}{3} = y^2 \qquad 64 = y^2 \qquad \boxed{y = 8}$

$x = \frac{192}{8} = 24$

$\boxed{x = 24}$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



$4x+3y=200$

$4x=-3y+200$

$x=\frac{-3}{4}y+50$

$A=2xy \Longrightarrow A(y)=2\left(\frac{-3}{4}y+50\right)y$

$=\frac{-3}{2}y^2+100y$

$A'(y)=-3y+100$

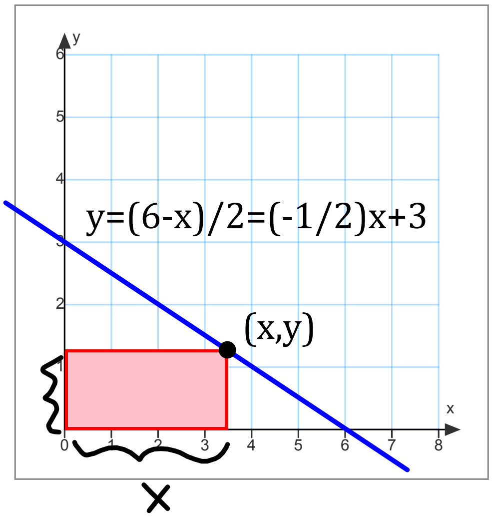
$-3y+100=0$

$y=\frac{100}{3}$ ft

$x=\frac{-3}{4}\left(\frac{100}{3}\right)+50$

$x=25$ ft

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$A(x)=x\left[-\frac{1}{2}x+3\right]$

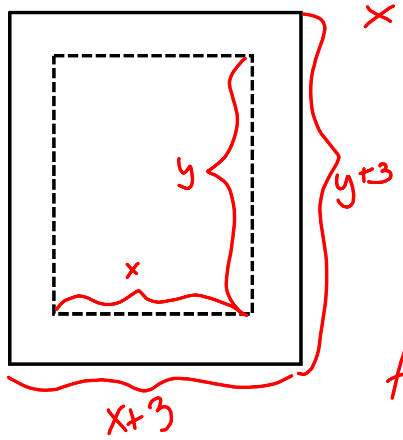
$=-\frac{1}{2}x^2+3x$

$A'(x)=-x+3$

$-x+3=0$

$x=3$ $y=\frac{3}{2}$

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



$xy = 36 \qquad y = \frac{36}{x}$

$A(x,y) = (x+3)(y+3)$

$A(x) = (x+3)\left(\frac{36}{x}+3\right)$

$= 36 + 3x + 3\cdot\frac{36}{x} + 9$

$A'(x) = 3 - \frac{3\cdot 36}{x^2}$

$3 - \frac{3\cdot 36}{x^2} = 0$

$36 = x^2$

$6 = x$

Actual paper dimensions: 9" x 9"

Homework #9

(due Fri, 10/17)

- 3.5 #15-31odd - limits at infinity
- 3.7 #3,5,17,23,29 - optimization
- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs

Wed, 10/8 - conceptual multiple choice quiz to earn bonus points on Test 3

Fri - 10/10 - no class!