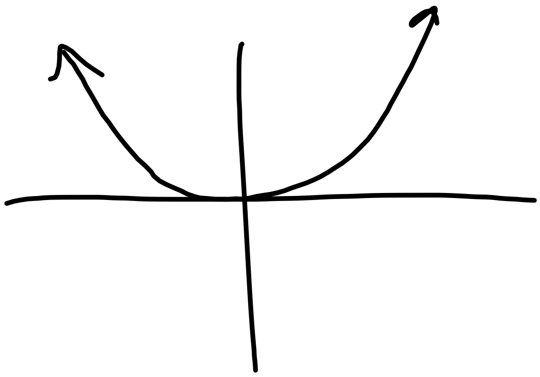
differentiability implies continuity

$f$ is diff. on $[a,b]$

$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$



# 7.7 Indeterminate Forms & L'Hôpital's Rule

$\frac{0}{0}, \ \frac{\infty}{\infty}, \ 0 \cdot \infty, \ 1^{\infty}, \ 0^0, \ and \ \infty - \infty$ are called indeterminate forms.

**L'Hôpital's Rule:**

Let $f$ and $g$ be functions that are differentiable on an open interval $(a, b)$ containing $c$, except possibly at $c$ itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, $\infty/\infty$, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

7.7

12. $$\lim_{x\to -1} \frac{x^2-x-2}{x+1} = \lim_{x\to -1} \frac{2x-1}{1} = \boxed{-3}$$

$$= \lim_{x\to -1} \frac{(x-2)(x+1)}{x+1}$$

$$= \lim_{x\to -1} (x-2) = -3$$

16. $$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3}$$

$$= \lim_{x\to 0^+} \frac{e^x-1}{3x^2}$$

$$= \lim_{x\to 0^+} \frac{e^x}{6x} = \boxed{+\infty}$$

18. $$\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\frac{1}{x^2} \cdot 2x}{2x}$$

$$= \lim_{x \to 1} \frac{1}{x^2}$$

$$= \boxed{1}$$

20. $$\lim_{x \to 0} \frac{\sin ax}{\sin bx}$$

$$= \lim_{x \to 0} \frac{a \cdot \cos ax}{b \cdot \cos bx}$$

$$= \boxed{\frac{a}{b}}$$

28. $$\lim_{x\to\infty} \frac{x^2}{e^x}$$

$$= \lim_{x\to\infty} \frac{2x}{e^x}$$

$$= \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$$

36. $$\lim_{x\to\infty} \frac{e^{x/2}}{x}$$

$$\frac{x}{2} = \frac{1}{2}\cdot x$$

$$= \lim_{x\to\infty} \frac{\frac{1}{2}\cdot e^{x/2}}{1} = \boxed{\infty}$$

38. $\lim_{x\to 0^+} x^3 \cot x$

$= \lim_{x\to 0^+} \frac{x^3}{\tan x}$

$= \lim_{x\to 0^+} \frac{3x^2}{\sec^2 x} = \boxed{0}$

40. $\lim_{x\to\infty} x \tan\frac{1}{x}$ $\left(\frac{1}{x}\right)' = (x^{-1})' = -x^{-2} = \frac{-1}{x^2}$

$= \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\left(\sec^2\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \sec^2\frac{1}{x} = \boxed{1}$

42. $\lim_{x\to 0^+} (e^x + x)^{2/x}$

$y = \lim_{x\to 0^+} (e^x + x)^{2/x}$

$\ln y = \ln \left[\lim_{x\to 0^+} (e^x + x)^{2/x}\right]$

$\ln y = \lim_{x\to 0^+} \ln (e^x + x)^{2/x}$

$\ln y = \lim_{x\to 0^+} \frac{2}{x} \ln (e^x + x)$

$\ln y = \lim_{x\to 0^+} \frac{\ln (e^x + x)}{\frac{x}{2}}$

$\ln y = \lim_{x\to 0^+} \frac{\frac{1}{e^x + x} \cdot (e^x + 1)}{\frac{1}{2}}$

$\ln y = \lim_{x\to 0^+} \frac{2(e^x + 1)}{e^x + x}$

$\ln y = 4$

$e^{\ln y} = e^4$

$\boxed{y = e^4}$

44. $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x$

$y = \lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x$

$\ln y = \lim_{x\to\infty} x \ln\left(1 + \frac{1}{x}\right)$

$\ln y = \lim_{x\to\infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}}$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1 + \frac{1}{x}} \cdot \left(\frac{-1}{x^2}\right)}{\frac{-1}{x^2}}$

$\ln y = \lim_{x\to\infty} \frac{1}{1 + \frac{1}{x}}$

$\ln y = 1$

$y = \boxed{e}$

$$\lim_{x\to 0^+} (\sin x)^x$$

$\ln[f(x)]^p = p \cdot \ln[f(x)]$

$$y = \lim_{x\to 0^+} (\sin x)^x$$

$$\ln y = \lim_{x\to 0^+} x \ln(\sin x)$$

$$\ln y = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}}$$

$$\ln y = \lim_{x\to 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{\frac{-1}{x^2}}$$

$$\ln y = \lim_{x\to 0^+} (-x^2 \cot x)$$

$$\ln y = \lim_{x\to 0^+} \frac{-x^2}{\tan x}$$

$$\ln y = \lim_{x\to 0^+} \frac{-2x}{\sec^2 x}$$

$$\ln y = 0$$

$\log_a b = c \iff a^c = b$

$$e^{\ln y} = e^0$$

$$y = \boxed{1}$$

# Homework #9

**(due Fri, 10/17)**

- 3.5 #15-31odd - limits at infinity
- 3.7 #3,5,17,23,29 - optimization
- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs