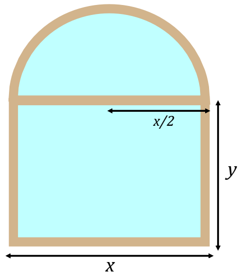
3.7 #23 - A **Norman window** is constructed by adjoining a semi-circle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.



$16 = x + 2y + \pi \cdot \frac{x}{2}$

$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$2y = 16 - x - \frac{\pi}{2}x$

$y = 8 - \frac{1}{2}x - \frac{\pi}{4}x$

$A(x) = \left(8 - \frac{1}{2}x - \frac{\pi}{4}x\right)x + \frac{\pi}{2} \cdot \frac{x^2}{4}$

$= 8x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$

$A(x) = 8x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$

$A'(x) = 8 - x - \frac{\pi}{4}x$

$8 - x - \frac{\pi}{4}x = 0$

$8 = x + \frac{\pi}{4}x$

$8 = x\left(1 + \frac{\pi}{4}\right)$

$x = \frac{8}{1+\frac{\pi}{4}} = \frac{8}{\frac{4+\pi}{4}}$

$\boxed{x = \frac{32}{4+\pi}}$

$y = 8 - \frac{1}{2}\left(\frac{32}{4+\pi}\right) - \frac{\pi}{4}\left(\frac{32}{4+\pi}\right)$

$= \frac{8(4+\pi)}{4+\pi} - \frac{16}{4+\pi} - \frac{8\pi}{4+\pi}$

$= \frac{32 + 8\pi - 16 - 8\pi}{4+\pi} = \boxed{\frac{16}{4+\pi} = y}$

$\lim_{x \to 0^+} x^{1/x} = 0^{\infty}$

$= \boxed{0}$

$1^{1/1} = 1, \quad \frac{1}{2}^{1/\frac{1}{2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}, \quad \frac{1}{3}^{1/\frac{1}{3}} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$0 \cdot \infty$

$\frac{\infty}{\infty}$

$\frac{0}{0}$

$\infty - \infty$

1^{∞}

0^{0}

49. $\lim_{x \to 1^+} (\ln x)^{x-1} \approx 0^0$

$y = \lim_{x \to 1^+} (\ln x)^{x-1}$

$\ln y = \ln \left[\lim_{x \to 1^+} (\ln x)^{x-1} \right]$

$\ln y = \lim_{x \to 1^+} \left[\ln (\ln x)^{x-1} \right]$

$\ln y = \lim_{x \to 1^+} \left[(x-1) \ln (\ln x) \right]$

$\ln y = \lim_{x \to 1^+} \left[\dfrac{\ln(\ln x)}{\frac{1}{x-1}} \right]$

$\ln y = \lim_{x \to 1^+} \dfrac{\frac{1}{\ln x} \cdot \frac{1}{x}}{\frac{-1}{(x-1)^2}}$

$\ln y = \lim_{x \to 1^+} \dfrac{-(x-1)^2}{x \ln x}$

$\ln y = \lim_{x \to 1^+} \dfrac{-2(x-1)}{1 \cdot \ln x + x \cdot \frac{1}{x}}$

$\ln y = \lim_{x \to 1^+} \dfrac{-2x+2}{\ln x + 1}$

$\ln y = 0$

$e^0 = y$ $\boxed{y = 1}$

For $0 \cdot \infty$ case, bring one to denominator to form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ case

Find the differential dy.

1. $y = x(1 - \cos x)$
2. $y = \sqrt{36 - x^2}$ $= (36 - x^2)^{1/2}$

1. $dy = \left[(1 - \cos x) + x(\sin x) \right] dx$

2. $dy = \dfrac{-x\,dx}{\sqrt{36 - x^2}}$ $=$ $dy = \frac{1}{2}(36 - x^2)^{-1/2} \cdot (-2x) \cdot dx$

3. Use differentials to approximate the value of the expression.

$\sqrt[3]{63}$ $f(x)=\sqrt[3]{x}$; $c=64$

$f(c)=\sqrt[3]{64}=4$

$f(c+\Delta x)\approx f(c)+f'(c)\cdot\Delta x$

$f'(x)=\dfrac{1}{3\sqrt[3]{x^2}}$ $f'(c)=\dfrac{1}{3\cdot 16}=\dfrac{1}{48}$

$\sqrt[3]{64+(-1)}\approx 4+\dfrac{1}{48}(-1)$

$=\dfrac{191}{48}$

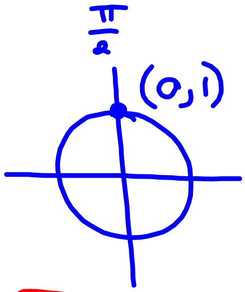
$\cos\dfrac{19\pi}{40}$ $c=\dfrac{\pi}{2}=\dfrac{20\pi}{40}$

$f(x)=\cos x$

$f(c)=\cos\dfrac{\pi}{2}=0$

$f'(x)=-\sin x$

$f'(c)=-\sin\dfrac{\pi}{2}=-1$



$\cos\left(\dfrac{20\pi}{40}-\dfrac{\pi}{40}\right)\approx 0+(-1)\cdot\left(\dfrac{-\pi}{40}\right)=\boxed{\dfrac{\pi}{40}}$

$\Delta x=-\dfrac{\pi}{40}$

$\lim_{x\to 0}\dfrac{e^x-(1-x)}{x}=\dfrac{0}{0}=\lim_{x\to 0}\dfrac{e^x+1}{1}=\dfrac{1+1}{1}=\boxed{2}$

$\lim_{x\to\infty}\dfrac{x^2}{\sqrt{x^2+1}}=\lim_{x\to\infty}\dfrac{x^2}{\sqrt{x^2}}=\lim_{x\to\infty}\dfrac{x^2}{|x|}=\lim_{x\to\infty}\dfrac{x^2}{x}=$

$=\lim_{x\to\infty}x=\boxed{\infty}$

$\lim_{x\to\infty}\dfrac{\sin x}{x-\pi}$

$\lim_{x\to\infty}\dfrac{\ln x^4}{x^3}$

Homework #9 (Fri, 10/17)

- 3.5 #15-31odd - limits at infinity
- 3.7 #3,5,17,23,29 - optimization
- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs

HW Due Thurs 10/23:
3.9 #5, 9; 11-19 odd; 45, 49

Test #4: Thurs 10/23
primarily on sections 3.5,3.7,and 7.7, with some review and 1 or 2 questions from 3.9

HW Due Fri 10/24:
handouts

Exam: Wed 10/29
- All A's on all 4 tests (after bonus pts) --> exempt from final (unless you want to take it because HW and quizzes are keeping you from an A in the class)
- Lowest of 4 regular test grades will be dropped (if it helps you)
- Final Exam can replace 2nd lowest test grade (if it helps you)