Turn in Homework #1
1.2 #1-7odd,9-18all

$|2| = 2$

$|-2| = 2 = -(-2)$

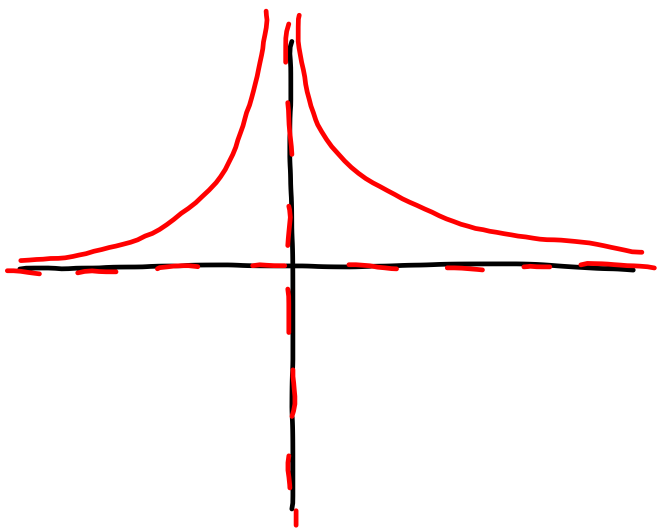
$$\lim_{x\to 3} \frac{|x-3|}{x-3}$$ does not exist

$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x-3>0 \\ & x>3 \\ \frac{-(x-3)}{x-3} = -1, & x-3<0 \\ & x<3 \end{cases}$$

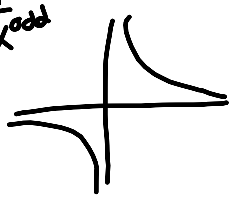
$\lim_{x\to 3^+} = 1$

$\lim_{x\to 3^-} = -1$

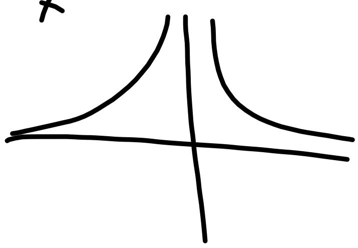
$$\lim_{x\to 0} \frac{1}{x^4} = \infty$$



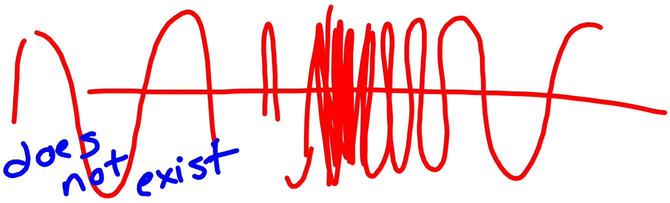
$y = \frac{1}{x^{odd}}$



$y = \frac{1}{x^{even}}$

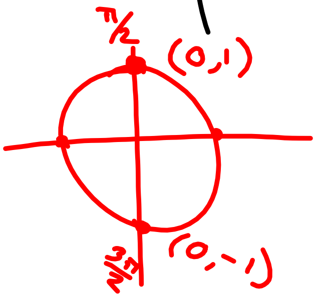


$$\lim_{x\to 0} \sin\frac{1}{x}$$



does not exist

x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin\frac{1}{x}$	1	-1	1	-1	1	-1



"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

$\lim_{x\to c} f(x)$ does not exist for <u>any</u> value of c

Graph the rational function.

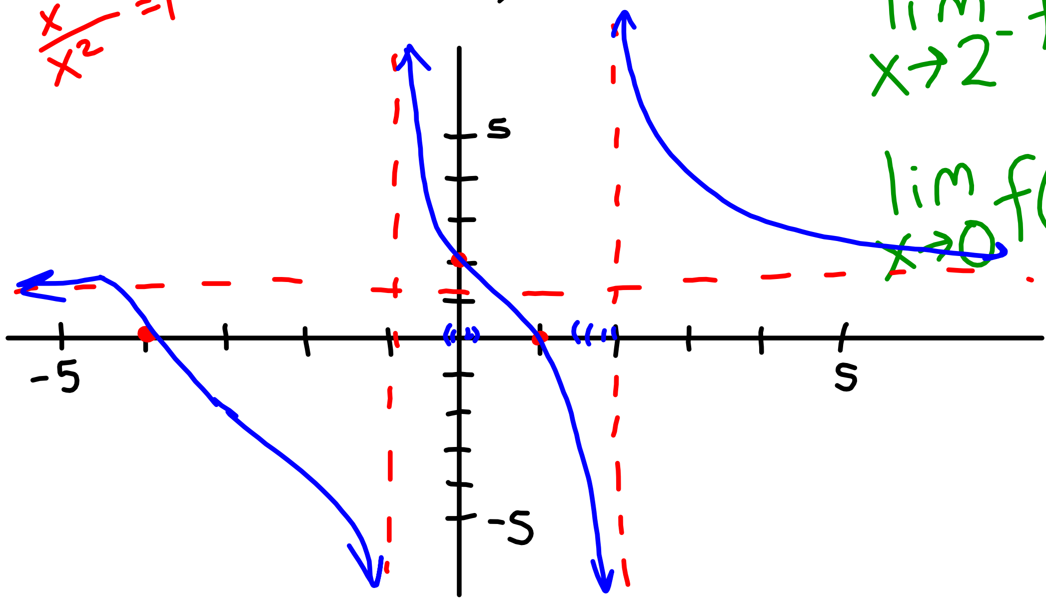
$$f(x) = \frac{(x+4)(x-1)}{(x-2)(x+1)}$$

$\frac{x^2}{x^2} = 1$

$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 0} f(x) = 2$

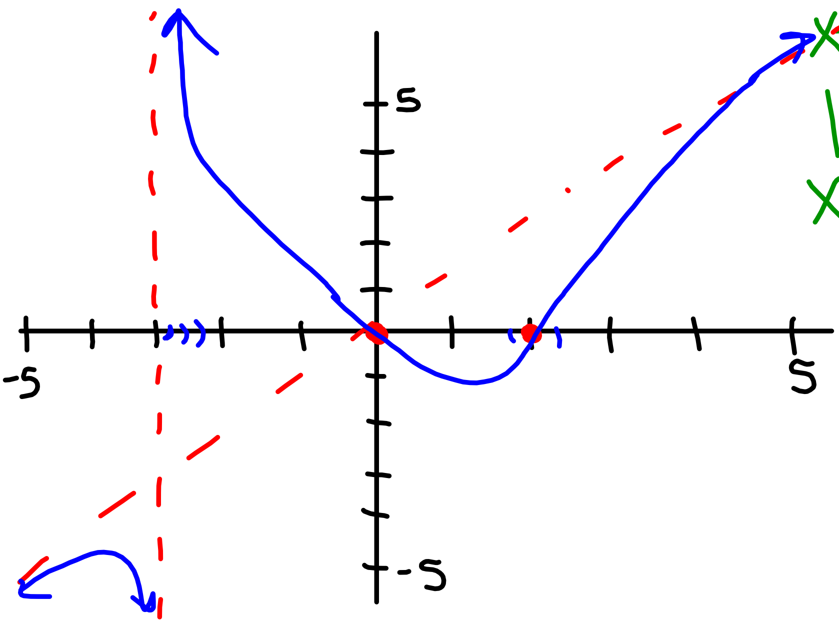


$$f(x) = \frac{x(x-2)}{x+3} \approx \frac{x^2}{x} = x$$

$\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

$\lim_{x\to 2} f(x) = 0$



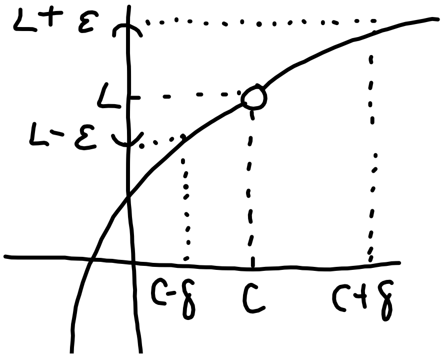
ε = epsilon

δ = delta

Building up to the $\epsilon - \delta$ Definition of the Limit

Translating the "informal description": $\lim_{x \to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L - \varepsilon, L + \varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c - \delta, c)$ or $(c, c + \delta)$.

$0 < |x - c| < \delta$

The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

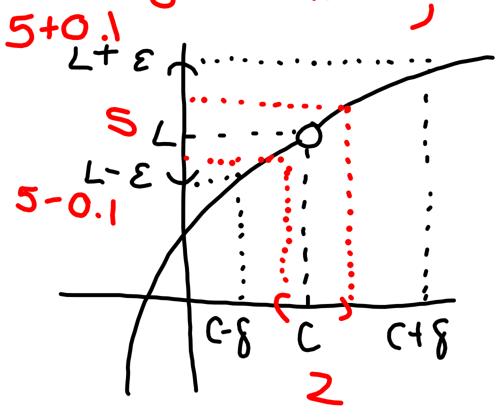
arbitrarily small # that we choose

$$\lim_{x \to c} f(x) = L$$

another arbitrarily small # dependent on the ε we chose

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

x is δ-close to c, then f(x) is ε-close to L.

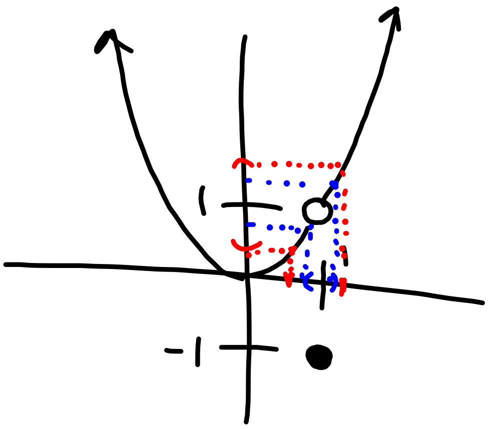


$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$



$$\lim_{x \to 1} f(x) = 1$$

$c = 1$ $L = 1$

HW #1 (submitted 11/7):
1.2 #1-7odd,9-18all

HW #2 (due 11/14):
1.2 #23, 25, 27, 29, 30, 31
and watch all of the Khan Academy epsilon-delta videos!