Like Math? Trying to earn a Concentration or Distinction in Math? Want an advantage on the PSAT, ACT, and SAT? Join **Problem Solving**! The class meets on Mondays and Wednesdays at 5:00 in S205. DR forms must get the proper signatures and be submitted no later than this Friday, so see Mrs. Prokhorova ASAP if you are interested!

Math Lab officially starts Monday! Think you might need help this week? Come to my Office Hours on Thursday afternoon (2:45-4:40) to ask questions or just to work on your homework in a structured environment.

Origami (and all other) **DR** forms must be submitted no later than this Friday!

Review:

What is the formula for the slope of the secant line to a function f through the points (x, f(x)) and (x+h, f(x+h))?

$$\frac{f(x+h)-f(x)}{x+h-x}$$

What is the formula for the slope of the tangent line to a function f through the point (x, f(x))?

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

When is $|x|=-x$? $x<0$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x}{\Delta x}$$

Graph the rational function.

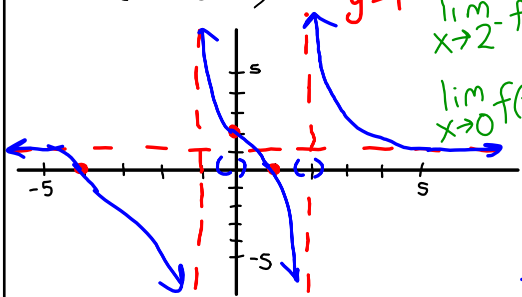
$$f(x)=\frac{(x+4)(x-1)}{(x-2)(x+1)}\approx\frac{x^2}{x^2}=1$$

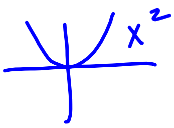
$y=1$

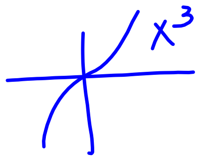
$$\lim_{x\to\infty}f(x)=1$$

$$\lim_{x\to 2^-}f(x)=-\infty$$

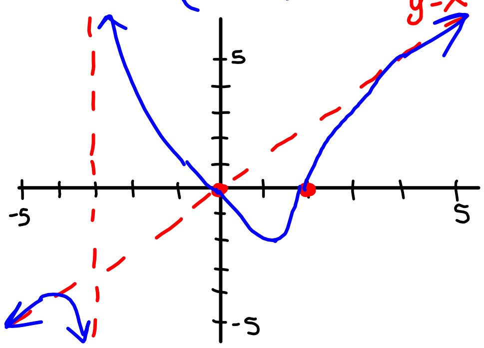
$$\lim_{x\to 0}f(x)=2$$







$$f(x) = \frac{(x)^1(x-2)^1}{(x+3)^1} \approx \frac{x^2}{x} = x$$



$$\lim_{x \to -\infty} f(x) = -\infty$$

$$\lim_{x \to -3^+} f(x) = \infty$$

$$\lim_{x \to 2} f(x) = 0$$

Informal Description of the Limit

y-values

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

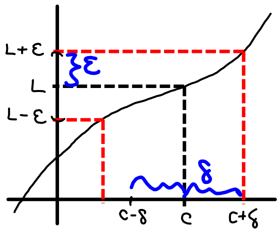
Building up to the $\epsilon - \delta$ Definition of the Limit

ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x\to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$|a-b|$ = distance between a & b

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$0 < |x - c| < \delta$

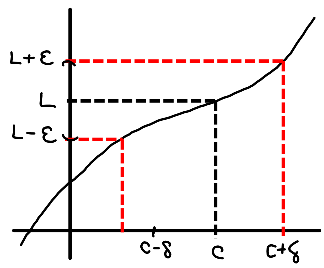
The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x\to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.



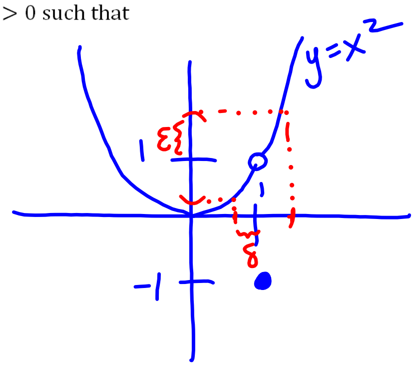
$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

$$\lim_{\underset{\uparrow}{\underset{c}{x\to 1}}} f(x) = \underset{\uparrow L}{1}$$



$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = 2x - 1$ $\quad c = 4 \quad L = 2(4) - 1 = 7$

Find $\lim_{x\to 4} f(x)$ and prove that is the limit using the ε-δ definition.

Let $\varepsilon > 0$ be given.

$|f(x) - L| = |2x - 1 - 7| = |2x - 8| = |2(x - 4)|$

$= 2|x - 4| < \varepsilon$

$\Rightarrow |x - 4| < \varepsilon/2$

Let $\boxed{\delta = \varepsilon/2}$

Then whenever $|x - c| < \delta$ $(|x - 4| < \delta)$,

we have $|f(x) - L| = |2x - 1 - 7| = 2|x - 4| < 2\cdot\delta$

$2\delta = 2\,\varepsilon/2 = \varepsilon$, i.e. $|f(x) - L| < \varepsilon$

Hence, $\lim_{x\to 4} f(x) = 7$

$\varepsilon-\delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x+3$; find $\lim_{x\to 1} f(x)$ & find a δ.

$c = 1$

$L = -5(1)+3 = \boxed{-2}$

$|f(x) - L| = |-5x+3-(-2)| = |-5x+5| = \ldots$

$\ldots = |-5(x-1)| = |-5||x-1| = 5|x-1|$

$5|x-1| < \varepsilon$

$|x-1| < \varepsilon/5$

$\boxed{\delta = \varepsilon/5}$

Find δ for $\varepsilon = 0.01$

$c = 4$; $L = 4 - \frac{4}{2} = 4-2 = 2$

24. $\lim_{x\to 4} \left(4 - \frac{x}{2}\right)$

$|f(x) - L| = |4 - \frac{x}{2} - 2| = |-\frac{x}{2} + 2| = |-\frac{1}{2}(x-4)|$

$= |-\frac{1}{2}||x-4| = \frac{1}{2}|x-4| < 0.01$

$\Rightarrow |x-4| < \boxed{0.02 = \delta}$

Homework for Test #1:

- 1.2 #1-7odd,9-18all — limits from graphs
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
  (and watch all of the Khan Academy epsilon-delta videos!)

- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically
  1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem

- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity
- 1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems
  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits
- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems handout

(not due until after the test, but will still help you with limits that will be on the test)

- 2.1 -#1-23odd — derivative definition