Homework for Test #1:

- 1.2 #1-7odd,9-18all — limits from graphs
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
  (and watch all of the Khan Academy epsilon-delta videos!)

Tues

- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically
  1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem

- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity
- 1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems
  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits
- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems handout

(not due until after the test, but will still help you with limits that will be on the test)

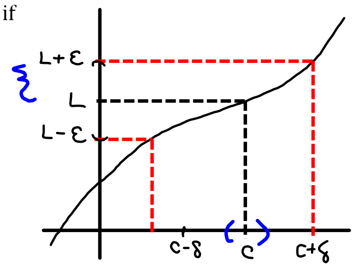
- 2.1 -#1-23odd — derivative definition

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.



$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

Prove that the limit is L using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x\to -3}(2x+5) = 2(-3)+5 = -6+5 = -1$

$c = -3$ $\boxed{L=-1}$

Let $\varepsilon > 0$ be given.

$|f(x) - L| = |2x+5-(-1)| = |2x+6| = 2|x-(-3)|$

$2|x-(-3)| < \varepsilon$

$|x-(-3)| < \varepsilon/2$ Take $\delta = \varepsilon/2$.

Then whenever $|x-c| < \delta$ or $|x-(-3)| < \delta$,

we have $|f(x) - L| = |2x+5-(-1)| = 2|x-(-3)|$

$2|x-(-3)| < 2\delta = 2\cdot\varepsilon/2 = \varepsilon$

i.e. If $|x-c| < \delta$, then $|f(x)-L| < \varepsilon$

Hence $\lim_{x\to -3}(2x+5) = -1$.

$f(x) = -5x+3$; find $\lim_{x\to 1} f(x)$ & find a δ.

$c = 1$

$L = -5(1)+3 = \boxed{-2}$

$|f(x) - L| = |-5x+3-(-2)| = |-5x+5| = |-5(x-1)|$

$= 5|x-1| < \varepsilon$

$|x-1| < \boxed{\varepsilon/5 = \delta}$

Find δ for $\varepsilon = 0.01$

26. $\lim_{x \to 5}(x^2 + 4) = 5^2 + 4 = 29$

$c = 5$ $\quad \boxed{L = 29}$

$|f(x) - L| = |x^2 + 4 - 29| = |x^2 - 25| = |(x+5)(x-5)|$

Since $x \to 5$, $\quad 4 < x < 6$

$4 + 5 < x + 5 < 6 + 5$

$9 < x + 5 < 11$

$|(x+5)(x-5)| < 11\,|x-5| < 0.01$

$|x-5| < 0.01/11 = \delta$

1.3 Evaluating Limits Analytically

If $\lim_{x \to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, f, g *– functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant $\lim_{x\to c} b = b$
2. Identity $\lim_{x\to c} x = c$
3. Polynomial $\lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x\to c}[f(x)g(x)] = LK$
7. Quotient $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x\to c}[f(x)]^n = L^n$

$\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

$\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right] \cdot \left[\lim_{x\to c} g(x)\right]$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, $\lim_{x\to c} g(x) \neq 0$

$\lim_{x\to c}[f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

$\lim_{x\to c} a = a$

$\lim_{x\to c} x = c$

$\lim_{x\to c} x^n = c^n$

$\lim_{x\to 5} (-3) = -3$

$\lim_{x\to -\pi} x = -\pi$

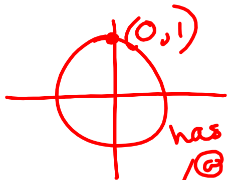
$\lim_{x\to -1} x^5 = (-1)^5 = \boxed{-1}$

1.3

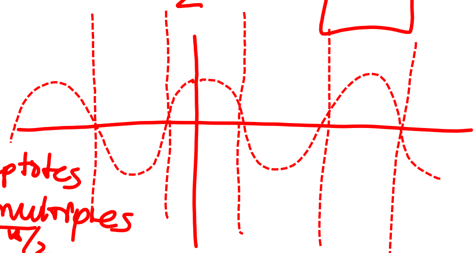
12. $\lim_{x\to 1}(3x^3-2x^2+4) = 3(1)^3-2(1)^2+4$
$= 3-2+4 = \boxed{5}$

18. $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \frac{2}{-1} = \boxed{-2}$

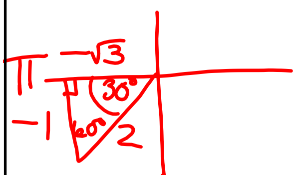
30. $\lim_{x\to 1}\sin\frac{\pi x}{2} = \sin\frac{\pi(1)}{2} = \boxed{1}$



has asymptotes @ odd multiples of $\pi/2$



36. $\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right) = \sec\frac{7\pi}{6} = \boxed{-\frac{2}{\sqrt{3}}}$



38. $\lim_{x\to c} f(x) = \frac{3}{2}$; $\lim_{x\to c} g(x) = \frac{1}{2}$

(a) $\lim_{x\to c} [4f(x)] = 4 \cdot \lim_{x\to c} f(x) = 4 \cdot \frac{3}{2} = \boxed{6}$

(b) $\lim_{x\to c} [f(x)+g(x)] = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = \boxed{2}$

(c) $\lim_{x\to c} [f(x)g(x)] = \frac{3}{2} \cdot \frac{1}{2} = \boxed{\frac{3}{4}}$

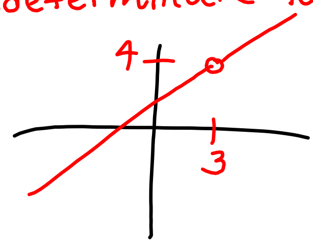
(d) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2} \cdot \frac{2}{1} = 3$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \frac{3^2-2(3)-3}{3-3} = \frac{0}{0}$$ oh no!

indeterminate form

$$= \lim_{x\to 3} \frac{(x+1)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x\to 3} (x+1) = 3+1 = \boxed{4}$$



$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0}$$

$(a+b)(a-b) = a^2 - b^2$

multiply by conjugate

$$= \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \lim_{x\to 4} (\sqrt{x}+2) = \sqrt{4}+2 = 2+2 = \boxed{4}$$

Given $f(x) = 2x^2 + 3x + 1$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$f(x+h) = 2(x+h)^2 + 3(x+h) + 1$

$$= \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2+3x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h + \cancel{1} - \cancel{2x^2} - \cancel{3x} - \cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}} = 4x + 2(0) + 3$$

$$= \boxed{4x+3}$$

$f(x) = x^3$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - (\cancel{x^3})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = 3x^2 + 3x(0) + 0^2$$

$$= \boxed{3x^2}$$