Homework for Test #1:

- **1.2 #1-7odd,9-18all** — **limits from graphs**
- **1.2 #23, 25, 27, 29, 30, 31** — **epsilon delta**
  (and watch all of the Khan Academy epsilon-delta videos!)
- **1.3 #11,17,27-35odd, 39-61odd** — **evaluating limits analytically**
  } Tuesday
  1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem
- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity
- 1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems
  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits
- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems handout

(not due until after the test, but will still help you with limits that will be on the test)

- 2.1 -#1-23odd — derivative definition

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant $\lim_{x\to c} b = b$
2. Identity $\lim_{x\to c} x = c$
3. Polynomial $\lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x\to c}[f(x)g(x)] = LK$
7. Quotient $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x\to c}[f(x)]^n = L^n$

$\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

$\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right] \cdot \left[\lim_{x\to c} g(x)\right]$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, $\lim_{x\to c} g(x) \neq 0$

$\lim_{x\to c} [f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

1.3 Evaluating Limits Analytically

42. $h(x) = \dfrac{x^2 - 3x}{x}$

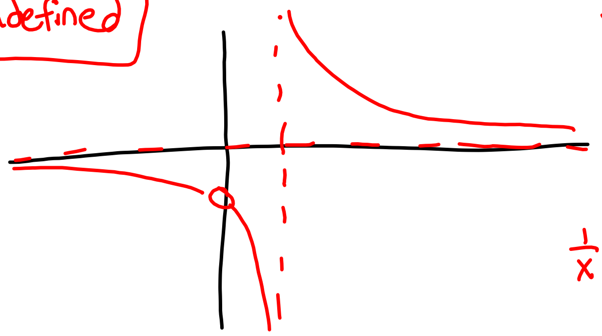
(a) $\lim\limits_{x \to -2} h(x) = \dfrac{(-2)^2 - 3(-2)}{-2} = \dfrac{4+6}{-2} = \dfrac{10}{-2} = \boxed{-5}$

(b) $\lim\limits_{x \to 0} h(x)$

$= \lim\limits_{x \to 0} \dfrac{\cancel{x}(x-3)}{\cancel{x}} = 0 - 3 = \boxed{-3}$

44. $\lim\limits_{x \to 1} \dfrac{x}{x^2 - x} = \lim\limits_{x \to 1} \dfrac{x}{x(x-1)} = \lim\limits_{x \to 1} \dfrac{1}{x-1}$

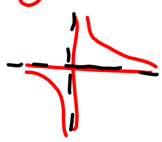
$\boxed{\text{undefined}}$



hole @ $x = 0$

asymptote @ $x = 1$

$\frac{1}{x}$



48. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}$

$= (-1)^2-(-1)+1$

$= 1+1+1 = \boxed{3}$

54. $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$(a-b)(a+b)$
$=a^2-b^2$

$= \lim_{x \to 0} \frac{(\cancel{2}+x)\cancel{-2}}{x(\sqrt{2+x}+\sqrt{2})}$

$= \lim_{x \to 0} \frac{1}{\sqrt{2+x}+\sqrt{2}} = \frac{1}{\sqrt{2+0}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$

58. $\lim_{x \to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{\frac{x}{1}} = \lim_{x \to 0} \left(\frac{1}{x+4} - \frac{1}{4} \right) \cdot \frac{1}{x}$

$= \lim_{x \to 0} \left(\frac{1}{x+4} \cdot \frac{4}{4} - \frac{1}{4} \cdot \frac{x+4}{x+4} \right) \cdot \frac{1}{x}$

$= \lim_{x \to 0} \left(\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)} \right) \cdot \frac{1}{x}$

$= \lim_{x \to 0} \frac{-\cancel{x}}{4(x+4)} \cdot \frac{1}{\cancel{x}} = \lim_{x \to 0} \frac{-1}{4(x+4)} = \boxed{-\frac{1}{16}}$

66. $\lim_{x \to 2} \dfrac{x^5 - 32}{x - 2}$

2	1	0	0	0	0	-32
	↓	2	4	8	16	32
	1	2	4	8	16	0

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x^4 + 2x^3 + 4x^2 + 8x + 16)}{\cancel{x-2}}$

$= 2^4 + 2(2)^3 + 4(2)^2 + 8(2) + 16 = \boxed{80}$

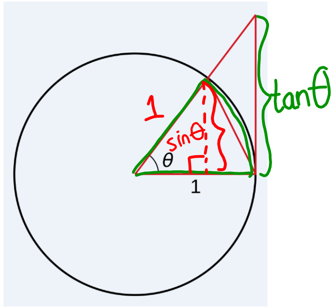
1.3 The Squeeze Theorem

$$\lim_{x\to 0}\frac{\sin x}{x} = ?$$

Area of whole circle $= \pi r^2|_{r=1} = \pi$

$$\frac{\text{Area of whole circle}}{\text{Total angle of circle}} = \frac{\text{Area of sector}}{\theta}$$

$$\frac{\pi}{2\pi} = \frac{\text{Area of sector}}{\theta} \rightarrow \text{Area of sector} = \frac{\theta}{2}$$



Area of outer triangle $\geq$ Area of sector $\geq$ Area of inner triangle

$$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$$

Multiply through by $\frac{2}{\sin\theta}$

$$\frac{\sin\theta}{2\cos\theta}\cdot\frac{2}{\sin\theta} \geq \frac{\theta}{2}\cdot\frac{2}{\sin\theta} \geq \frac{\sin\theta}{2}\cdot\frac{2}{\sin\theta}$$

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

Take reciprocals and reverse inequalities

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$$

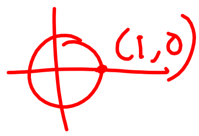
Take limits

$$\lim_{\theta\to 0}\cos\theta \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1$$

$$\cos 0 \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq 1$$

$$1 \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq 1$$

Therefore $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \boxed{1}$



$-1 \leq \sin x \leq 1$

$-1 \leq \cos x \leq 1$

The Squeeze Theorem:

If $f(x) \le g(x) \le h(x)$ *and* $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$,

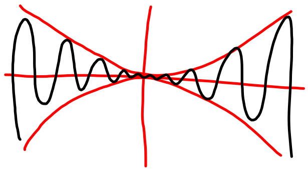
Then $\lim_{x\to c} g(x) = L$.

$-1 \le \sin x \le 1$

$-1 \le \cos x \le 1$

$-x^2 \le x^2 \sin x \le x^2$

$-1 \le \sin x \le 1$



Special Limits Derived by Squeeze Theorem:

$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$; $\boxed{\lim_{x\to 0} \frac{1-\cos x}{x} = 0}$

Memorize!!

Use the squeeze theorem to find

$\lim_{x\to 0}\left(x^2 \cos\frac{5}{x} - 3\right)$

$-1 \le \cos x \le 1$

$-1 \le \cos \frac{5}{x} \le 1$

$-x^2 \le x^2 \cos \frac{5}{x} \le x^2$

$-x^2 - 3 \le x^2 \cos \frac{5}{x} - 3 \le x^2 - 3$

$\lim_{x\to 0}(-x^2-3) \le \lim_{x\to 0}\left(x^2 \cos\frac{5}{x} - 3\right) \le \lim_{x\to 0}(x^2-3)$

$-3 \le \lim_{x\to 0}\left(x^2 \cos\frac{5}{x} - 3\right) \le -3$

Therefore, by the Squeeze Theorem,

$\lim_{x\to 0}\left(x^2 \cos\frac{5}{x} - 3\right) = \boxed{-3}$

68. $\lim_{x\to 0} \frac{3(1-\cos x)}{x}$

$= 3 \cdot \lim_{x\to 0} \frac{(1-\cos x)}{x}$

$= 3 \cdot 0$

$= \boxed{0}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

72. $\lim_{x\to 0} \frac{\tan^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{x}$

$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{\cos^2 x}$

$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \frac{\sin x}{\cos^2 x}\right)$

$= 1 \cdot \frac{\sin 0}{(\cos 0)^2} = 1 \cdot \frac{0}{1^2} = \boxed{0}$

$\frac{abc}{def} = \frac{a}{f} \cdot \frac{bc}{d} \cdot \frac{1}{e}$

78. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 2x}{1} \cdot \frac{1}{\sin 3x}$

$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{3x}{\sin 3x} \cdot \frac{2}{3}$

$= \left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right)\left(\lim_{x\to 0} \frac{1}{\frac{\sin 3x}{3x}}\right) \cdot \lim_{x\to 0} \frac{2}{3}$

$= 1 \cdot \frac{1}{1} \cdot \frac{2}{3} = \boxed{2/3}$

If $x \to 0$,
then $2x \to 0$ &
$3x \to 0$

$\frac{\sin 2x}{2x}$ & $\frac{\sin 3x}{3x}$
behave like $\frac{\sin x}{x}$