Homework for Test #1:

Test #1 - Tues. 9/1?

- **1.2 #1-7odd,9-18all** — **limits from graphs**
- **1.2 #23, 25, 27, 29, 30, 31** — **epsilon delta**
  (and watch all of the Khan Academy epsilon-delta videos!)
- **1.3 #11,17,27-35odd, 39-61odd** — **evaluating limits analytically**

Tuesday

  1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem

- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity
- 1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems

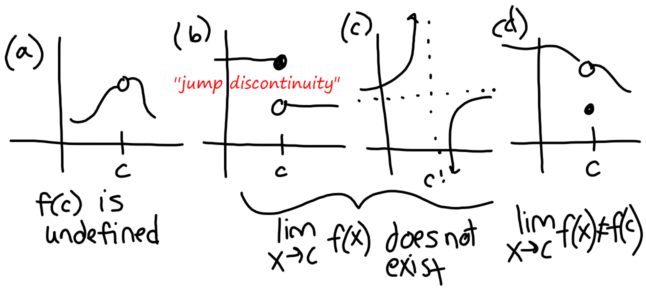
Fri

  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits
- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems handout

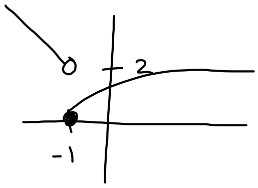
(not due until after the test, but will still help you with limits that will be on the test)

- 2.1 -#1-23odd — derivative definition

1.4 Continuity and One-Sided Limits



These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$\lim_{x \to -1^-} f(x) = 2$

$\lim_{x \to -1^+} f(x) = 0$

$\lim_{x \to -1} f(x) =$ does not exist/ undefined

One-Sided Limits

$\lim_{x \to c^+} f(x) = L \quad limit\ from\ the\ right$

$\lim_{x \to c^-} f(x) = L \quad limit\ from\ the\ left$

$$\boxed{\begin{gathered}\lim_{x \to c} f(x) = L \ \ if\ and\ only\ if \\ \lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)\end{gathered}}$$

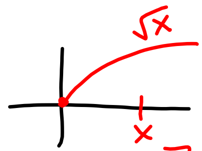
Continuity at a point

A function f is <u>continuous at c</u> if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

* $\lim_{x \to c} f(x) = f(c)$



[0, x]

[a, b]

Continuity on an open interval

A function is <u>continuous on an open interval</u> if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is <u>everywhere continuous</u>.

Continuity on a closed interval

A function f is <u>continuous on the closed interval</u> $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

10. $\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

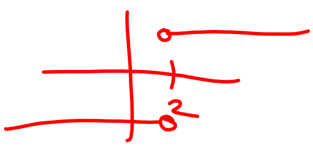
$= \lim_{x\to 4^-} \frac{1\ \cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)}$

$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

12. $\lim_{x\to 2^+} \frac{|x-2|}{x-2} = 1$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & x-2>0 \\ & x>2 \\ \frac{-(x-2)}{x-2} = -1, & x-2<0 \\ & x<2 \end{cases}$



1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)}$$

removable discontinuity @ $x=2$

non-removable discontinuity @ $x=-1$

f is continuous on $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

Discuss the [dis]continuity of the function.

$$f(x) = \frac{|x-2|}{x-2}$$

f has a non-removable (jump) discontinuity @ $x=2$

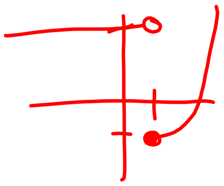
f is continuous on $(-\infty,2)\cup(2,\infty)$

$$f(x) = \begin{cases} x^2 - 2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

Discuss the [dis]continuity of the function.

$1^2 - 2 = 1 - 2 = -1$

$5 \neq -1$



f has a non-removable (jump) discontinuity @ $x = 1$

f is continuous on $(-\infty, 1) \cup [1, \infty)$

$$f(x) = \begin{cases} x+6, & x \leq -2 \\ x^2, & -2 < x \leq 3 \\ 8 & x > 3 \end{cases}$$

Discuss the [dis]continuity of the function.

$-2 + 6 = 4$

$(-2)^2 = 4$

$3^2 = 9$

$8 \neq 9$

non-removable (jump) discontinuity @ $x = 3$

f is continuous on $(-\infty, 3] \cup (3, \infty)$

$$f(x) = \begin{cases} \frac{|x-3|}{3-x}, & |x-3|>5 \\ x^2-3, & -2\le x\le 8 \end{cases}$$

Discuss the [dis]continuity of the function.

$x-3>5$ or $x-3<-5$

$x>8$ or $x<-2$

$$= \begin{cases} \frac{|x-3|}{3-x}, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ \frac{|x-3|}{3-x}, & x>8 \end{cases}$$

$$\frac{|x-3|}{3-x} = \begin{cases} \frac{x-3}{3-x} = -1, & x-3>0 \quad x>3 \\ \frac{-(x-3)}{3-x} = 1, & x-3<0 \quad x<3 \end{cases}$$

$$= \begin{cases} 1, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ -1, & x>8 \end{cases}$$

$(-2)^2-3 = 4-3 = 1$

cts @ -2

$8^2-3 \neq -1$ disc @ 8

non-removable discontinuity @ $x=8$

f is continuous on $(-\infty, 8] \cup (8, \infty)$