Homework for Test #1 Test #1 - Friday, 09/04

HW #1 Submitted Tues. 8/25:

- 1.2 #1-7odd,9-18all — limits from graphs
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
  (and watch all of the Khan Academy epsilon-delta videos!)
- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically

HW #2 Submitted Fri. 8/28:

- 1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem
- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity

HW #3 Due Tues. 9/01:

- 1.4 #**19**,21,23,**51**,**57**,**59**,**63**,**69**,**71** — misc. continuity problems
  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits

HW #4 Due Fri. 9/04:

- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems handout
- 2.1 -#1-23odd — derivative definition

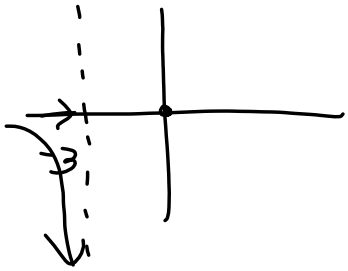
$$\lim_{\Delta x \to 0^-} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} \qquad f(x) = \frac{1}{x}$$

$$= \lim_{h \to 0^-} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0^-} \frac{x - (x+h)}{x(x+h)} \cdot \frac{1}{h} = \frac{-1}{x^2}$$

$$\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}} \cdot \frac{\sqrt{x^2-9}}{\sqrt{x^2-9}}$$

$$= \lim_{x\to -3^-} \frac{x\sqrt{x^2-9}}{x^2-9}$$

$$= -\infty$$



$$\lim_{t\to 0} \frac{\sin 3t}{2t}$$

$$= \lim_{t\to 0} \frac{\sin 3t}{3t} \cdot \frac{3}{2}$$

$$= \frac{3}{2} \cdot \lim_{t\to 0} \frac{\sin 3t}{3t}$$

$$= \frac{3}{2} \cdot 1 = \frac{3}{2}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to a}(b-|x-a|) \le f(x) \le (b+|x-a|)$$

$$\lim_{x\to a} f(x) = b \le \lim_{x\to a} f(x) \le b$$

$$\lim_{x\to a} f(x) = b$$

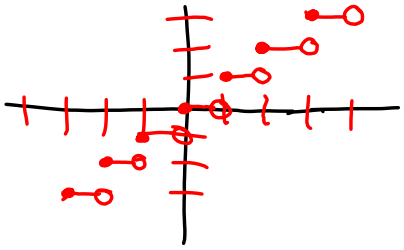
Discuss the [dis]continuity of the function.

The Greatest Integer Function

$[\![x]\!]$ = the greatest integer less than or equal to x

$\lfloor x \rfloor$ $\quad [\![\pi]\!] = 3 \quad [\![-2.63]\!] = -3$

$[\![-5]\!] = -5$



non-removable discontinuities @ all integers

continuous on all intervals of the form $[x, x+1), x \in \mathbb{Z}$

22. $\lim_{x \to 2^+} 2x - [\![x]\!]$

$= \lim_{x \to 2^+} 2x - \lim_{x \to 2^+} [\![x]\!]$

$= 4 - 2$

$= \boxed{2}$

24. $\lim_{x \to 1} \left(1 - [\![\frac{-x}{2}]\!]\right)$

$= \lim_{x \to 1} 1 - \lim_{x \to 1} [\![\frac{-x}{2}]\!]$

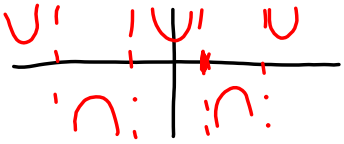
$= 1 - (-1)$

$= \boxed{2}$

20. $\lim_{x \to \frac{\pi}{2}} \sec x$

does not exist

$\lim_{x \to \frac{\pi}{2}^+} \sec x = -\infty$



sec x has vertical asymptotes @ odd multiples of $\pi/2$

52. $f(x) = \tan \frac{\pi x}{2}$

Discuss the [dis]continuity of the function.

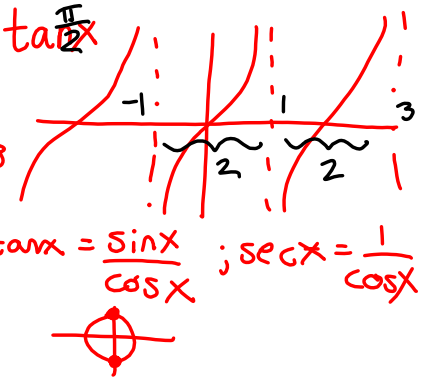
period: $\frac{\pi}{\pi/2} = \frac{\pi}{1} \cdot \frac{2}{\pi} = 2$

non-removable discontinuities @ all odd integers

continuous on all intervals of the form

$(2x-1, 2x+1)$, $x \in \mathbb{Z}$

[or: $(2x+1, 2x+3)$ etc.]



$\tan x = \frac{\sin x}{\cos x}$; $\sec x = \frac{1}{\cos x}$

62. $f(x)=\frac{1}{\sqrt{x}}, g(x)=x-1$

Discuss the continuity of f(g(x)).

$f(g(x))=\frac{1}{\sqrt{x-1}}$

f(g(x)) is continuous on its domain $\{x \mid x>1\} = (1,\infty)$

64. $f(x)=\sin x \; ; g(x)=x^2$

Discuss the continuity of f(g(x)).

$f(g(x)) = \sin(x^2)$

$\neq \sin x^2$

$(\sin x)^2 = \sin^2 x$

continuous on $(-\infty,\infty)$