Homework for Test #1

Test #1 - Friday, 09/04

HW #1 Submitted Tues. 8/25:

- 1.2 #1-7odd,9-18all — limits from graphs
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
  (and watch all of the Khan Academy epsilon-delta videos!)
- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically

HW #2 Submitted Fri. 8/28:

- 1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem
- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity

HW #3 Due Tues. 9/01:

- **1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems**
  1.4 #83,85 — intermediate value theorem
- **1.5 #1,3,25; 29-51odd — infinite limits**

HW #4 Due Fri. 9/04:

- Ch 1 review pp. 88-89 #3-49odd; 51-67odd
- Test #1 Practice Problems handout
- 2.1 -#1-23odd — derivative definition

Discuss the continuity of the following function. State which discontinuities are removable and which are non-removable, and state the intervals on which the function is continuous.

1. $f(x) = \frac{x-6}{x^2-7x+6}$ $= \frac{\cancel{x-6}}{(\cancel{x-6})(x-1)}$

non-removable discontinuity @ $x=1$
removable discontinuity @ $x=6$

f is continuous on $(-\infty,1)\cup(1,6)\cup(6,\infty)$

2. A function f is continuous at c if $\lim_{x\to c} f(x) =$ $f(c)$.

3. According to the Squeeze Theorem if $f(x) \le g(x) \le h(x)$, and $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$, then $\lim_{x\to c} g(x) =$ L.

4. State the epsilon-delta definition of the statement $\lim_{x\to c} f(x) = L$.

Given $\varepsilon > 0$, there exists $\delta > 0$ such that if $0 < |x-c| < \delta$, then $|f(x) - L| < \varepsilon$.

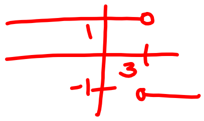
($0 < |x-c| < \delta$ implies $|f(x)-L| < \varepsilon$)

($|f(x)-L| < \varepsilon$ whenever $0 < |x-c| < \delta$)

Find the limits (if they exist). Draw a picture if this helps. Circle/box your final answers.

5. $\lim_{x\to 3} \frac{|x-3|}{3-x}$

does not exist



$$\frac{|x-3|}{3-x} = \begin{cases} \frac{x-3}{3-x} = -1, & x-3>0,\ x>3 \\ \frac{-(x-3)}{3-x} = 1, & x-3<0,\ x<3 \end{cases}$$

6. $\lim_{x\to 5} \frac{\sqrt{x-1}}{x-4}$

$= \frac{\sqrt{5-1}}{5-4} = \frac{\sqrt{4}}{1} = \boxed{2}$

7. $\lim_{x\to 0} \frac{\sin x}{x} = \boxed{1}$

8. $\lim_{x\to 0} \frac{1-\cos x}{x} = \boxed{0}$

Discuss the [dis]continuity of the function.

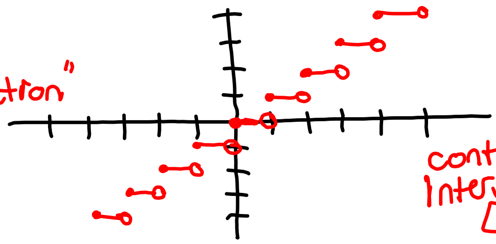
The Greatest Integer Function

$[\![x]\!]$ = the greatest integer less than or equal to x

$\lfloor x \rfloor$

$[\![\pi]\!] = 3$; $[\![-2.72]\!] = -3$

"step function"



non-removable discontinuities @ all integers

continuous on all intervals of the form $[x, x+1), x \in \mathbb{Z}$

22. $\lim_{x\to 2^+} 2x - [\![x]\!]$

$= \lim_{x\to 2^+} 2x - \lim_{x\to 2^+} [\![x]\!]$

$= 2(2) - 2$

$= 4 - 2$

$= \boxed{2}$

24. $\lim_{x\to 1} \left(1 - [\![-\frac{x}{2}]\!]\right)$

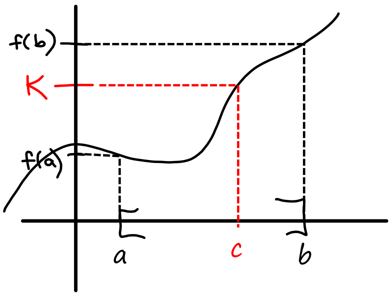
$= \lim_{x\to 1} 1 - \lim_{x\to 1} [\![-\frac{x}{2}]\!]$

$= 1 - (-1)$

$= \boxed{2}$

Intermediate Value Theorem

If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$.



Does the IVT guarantee a zero in the given interval?
x-value such that $f(x)=0$

76. $f(x)=x^3+3x-2$, $[0,1]$

$f(0)=0^3+3(0)-2=-2<0$
$f(1)=1^3+3(1)-2=2>0$

Yes, the IVT does guarantee a zero in $[0,1]$

Find it. Solve $x^3+3x-2=0$ for x
use solver function of calculator

84. $f(x) = x^2 - 6x + 8$; $[0, 3]$ $f(c) = 0$

$f(0) = 0^2 - 6(0) + 8 = 8 > 0$

$f(3) = 3^2 - 6(3) + 8 = -1 < 0$

Yes, the IVT guarantees a c in $[0, 3]$ such that $f(c) = 0$.

Find it. $x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\cancel{x = 4}$ $\boxed{x = 2}$

not in $[0,3]$

86. $f(x) = \dfrac{x^2 + x}{x - 1}$, $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$

$f\left(\frac{5}{2}\right) = \dfrac{\left(\frac{5}{2}\right)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \dfrac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \dfrac{35}{\cancel{4}_2} \cdot \dfrac{\cancel{2}}{3} = \dfrac{35}{6} < 6$

$f(4) = \dfrac{4^2 + 4}{4 - 1} = \dfrac{16 + 4}{4 - 1} = \dfrac{20}{3} > 6$

yes, the IVT guarantees a $c \in \left[\frac{5}{2}, 4\right]$ such that $f(c) = 6$

Find it: $\dfrac{x^2 + x}{x - 1} = 6$

$x^2 + x = 6(x - 1)$

$x^2 + x = 6x - 6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$\cancel{x = 2}$, $\boxed{x = 3}$

not in $\left[\frac{5}{2}, 4\right]$

1.5 Infinite Limits

$$\lim_{x \to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Find the vertical asymptotes (if any).

14. $f(x) = \frac{-4x}{x^2+4}$ ← denominator is never zero

$\boxed{\text{none}}$

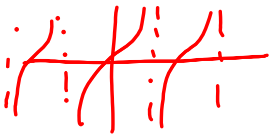
24. $h(x) = \frac{x^2-4}{x^3+2x^2+x+2} = \frac{(x-2)(x+2)}{(x^2+1)(x+2)}$

$x^2(x+2)+1(x+2)$

$\boxed{\text{none}}$

28. $g(\theta) = \frac{\tan\theta}{\theta}$ when $\theta = 0$, $\frac{\tan\theta}{\theta} = \frac{0}{0}$, indeterminate form

vertical asymptotes @ all odd multiples of $\pi/2$



Rules involving infinite limits

Let $\lim_{x\to c} f(x) = \infty$ *and* $\lim_{x\to c} g(x) = L$

1. $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

2. $\lim_{x\to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$

$\frac{1}{3}, \frac{1}{30}, \frac{1}{3000}, \frac{1}{3000000}, \ldots$

42. $\lim_{x\to 0^-}\left(x^2 - \frac{1}{x}\right) = \lim_{x\to 0^-} x^2 - \lim_{x\to 0^-} \frac{1}{x}$

$= 0^2 - (-\infty)$

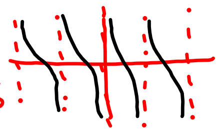
$= \boxed{\infty}$

$\frac{1}{-1/10}, \frac{1}{-1/1000}, \frac{1}{-\frac{1}{1000000}}, \ldots$

$-10 \quad -1000 \quad -1000000$

$\frac{c}{\pm\infty} \to 0 \; ; \; \frac{c}{\pm 0} \to \pm\infty$

46. $\lim_{x\to 0} \frac{x+2}{\cot x} \approx \frac{2}{\pm\infty}$



$= \lim_{x\to 0} (x+2)(\tan x)$

$= (0+2)(\tan 0)$

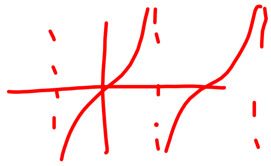
$= (2)(0) = \boxed{0}$

48. $\lim_{x\to\frac{1}{2}} x^2 \tan \pi x$

$= \left(\lim_{x\to\frac{1}{2}} x^2\right)\left(\lim_{x\to\frac{1}{2}} \tan \pi x\right)$

$= \frac{1}{4} \cdot (\pm\infty)$

$\boxed{\text{does not exist}}$



52. $\lim_{x\to 3^+} \sec \frac{\pi x}{6} = \boxed{-\infty}$