Homework for Test #1

Test #1 - Friday, 09/04

HW #1 Submitted Tues. 8/25:
- 1.2 #1-7odd,9-18all — limits from graphs
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
  (and watch all of the Khan Academy epsilon-delta videos!)
- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically

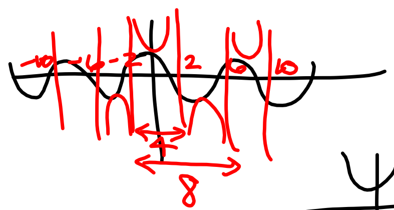
HW #2 Submitted Fri. 8/28:
- 1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem
- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity

HW #3 Due Tues. 9/01:
- **1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems**
  1.4 #83,85 — intermediate value theorem
- **1.5 #1,3,25; 29-51odd — infinite limits**

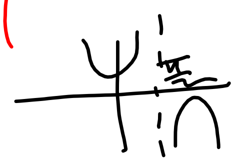
HW #4 Due Fri. 9/04:
- Ch 1 review pp. 88-89 #3-67odd
- Test #1 Practice Problems handout
- 2.1 -#1-23odd — derivative definition

$$\text{period} = \frac{\text{original } \left(\begin{array}{l}\pi \text{ for } \tan/\cot \\ 2\pi \text{ for } \sin, \cos, \sec, \csc\end{array}\right)}{x\text{-coeff.}}$$

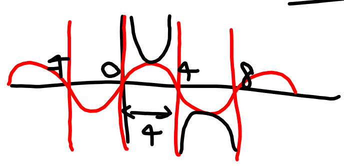
$\sec \frac{\pi}{4} x$



$$\frac{2\pi}{\pi/4} = 2\pi \cdot \frac{4}{\pi} = 8$$



$\csc \frac{\pi x}{4}$



$x^3 - x^2 + x - 2 = 4$

$x^3 - x^2 + x - 6 = 0$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ discriminant

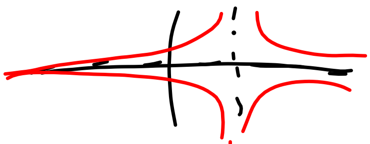
$(x-2)(x^2+x+3)=0$

no real zeros

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & \boxed{0} \end{array}$$

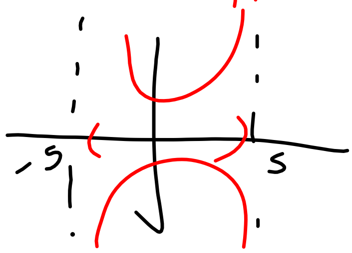
49. $\lim\limits_{x\to 1^+} \dfrac{x^2+x+1}{x^3-1} \approx \dfrac{1}{x}$

$= \boxed{\infty}$



51. $\lim\limits_{x\to 5^-} \dfrac{1}{x^2-25}$

$= \boxed{-\infty}$



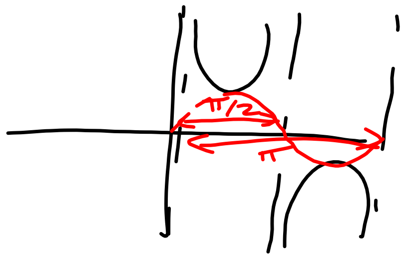
$$\lim_{x \to -3^-} \frac{x^2+2x-3}{x^2+x-6}$$

$$= \lim_{x \to -3^-} \frac{(\cancel{x+3})(x-1)}{(\cancel{x+3})(x-2)}$$

$$= \frac{-3-1}{-3-2}$$

$\csc 2x$

$$\text{period} = \frac{2\pi}{2} = \pi$$



2.1 The Derivative & The Tangent Line Problem

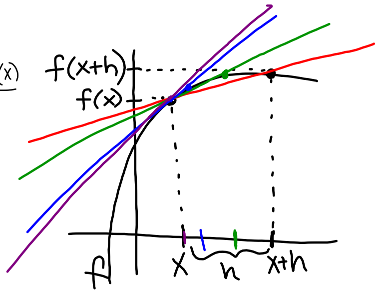
secant line
crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **<u>the derivative of f at x</u>**.

$$\boxed{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

	$\boldsymbol{f'(x)}$	"f $prime$ of x"
$\frac{d}{dx}[y]$	$\boldsymbol{\frac{dy}{dx}}$	"$derivative$ of y $with$ $respect$ to x"
$[f(x)]'$	$\boldsymbol{y'}$	"y $prime$"
$\frac{\partial f}{\partial x}$	$\boldsymbol{\frac{d}{dx}[f(x)]}$	"the $derivative$ $with$ $respect$ to x of $f(x)$"
	$\boldsymbol{D_x[y]}$	"the $partial$ $derivative$ $with$ $respect$ to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

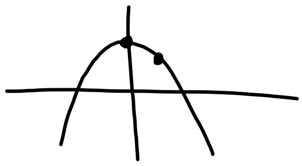
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = f'(c)$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

8. $g(x) = 5 - x^2$

find slope of tangent line at the points $(\overset{c}{2}, \overset{g(c)}{1})$ & $(\overset{c}{0}, \overset{g(c)}{5})$



$$m = g'(c) = \lim_{h \to 0} \frac{g(c+h) - g(c)}{h}$$

$$= \lim_{h \to 0} \frac{5 - (2+h)^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{5 - (4 + 4h + h^2) - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5} - \cancel{4} - 4h - h^2 - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-4-h)}{\cancel{h}} = -4 - 0 = \boxed{-4}$$

$(c, g(c)) = (0, 5)$

$$= \lim_{h \to 0} \frac{5 - (0+h)^2 - 5}{h}$$

$$= \lim_{h \to 0} \frac{5 - h^2 - 5}{h} = \lim_{h \to 0} \frac{-h^2}{h}$$

$$= \lim_{h \to 0} -h = -(0) = \boxed{0}$$

20. $f(x) = x^3 + x^2$

find the derivative

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 + (x+h)^2 - (x^3 + x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x^2} + 2xh + h^2 - \cancel{x^3} - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 2x + h)}{\cancel{h}}$$

$$= 3x^2 + 3x(0) + 0^2 + 2x + 0$$

$$= \boxed{3x^2 + 2x}$$

1. Find the limit , then use the $\varepsilon - \delta$ definition to prove that the limit is L .

$\lim_{x\to 8}(3x - 20) = 3(8) - 20 = 24 - 20 = 4$

$L = 4$; $c = 8$

$|f(x) - L| = \cdots = k|x - c| < \varepsilon$

$|x - c| < \boxed{\varepsilon/k}$ ← call that δ

$|3x - 20 - 4|$

$= |3x - 24| = 3|x - 8| < \varepsilon$

$|x - 8| < \boxed{\varepsilon/3 = \delta}$

2. Find the limit (if it exists).

$$\lim_{x \to -3} \frac{x^2 + 7x + 12}{x^2 - 9} = \lim_{x \to -3} \frac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}(x-3)}$$

$$= \frac{-3+4}{-3-3} = \boxed{-\frac{1}{6}}$$

3. Find the limit (if it exists).

$$\lim_{h \to 0} \frac{\overset{\neq f(x)+h}{f(x+h)} - f(x)}{h}, \quad \textit{where } f(x) = 5x^2 + 3$$

$$f(x+h) = 5(x+h)^2 + 3$$

$$= \lim_{h \to 0} \frac{5(x+h)^2 + 3 - (5x^2 + 3)}{h}$$

$$= \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) + 3 - 5x^2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 \cancel{+3} \cancel{-5x^2} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10x + 5h)}{\cancel{h}} = 10x + 5(0) = \boxed{10x}$$

4. Find the limit (if it exists).

$$\lim_{x\to 0} \frac{5\sin(2x)}{3x}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{5\cdot 2}{3}$$

$$= \left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right)\left(\lim_{x\to 0} \frac{10}{3}\right)$$

$$= 1 \cdot \frac{10}{3} = \boxed{\frac{10}{3}}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

$$\lim_{2x\to 0} \frac{\sin 2x}{2x} = 1$$

If $x\to 0$, $2x \to 0$