Homework for Test #1

Test #1 - Friday, 09/04

HW #1 Submitted Tues. 8/25:
- 1.2 #1-7odd,9-18all — limits from graphs
- 1.2 #23, 25, 27, 29, 30, 31 — epsilon delta
  (and watch all of the Khan Academy epsilon-delta videos!)
- 1.3 #11,17,27-35odd, 39-61odd — evaluating limits analytically

HW #2 Submitted Fri. 8/28:
- 1.3 #67-77odd; 87, 88 — limits with trig, squeeze theorem
- 1.4 #7-17odd; — limits of functions with discontinuities
  1.4 #25-28all; 39-47odd; — discuss (dis)continuity

HW #3 Due Tues. 9/01:
- 1.4 #19,21,23,51,57,59,63,69,71 — misc. continuity problems
  1.4 #83,85 — intermediate value theorem
- 1.5 #1,3,25; 29-51odd — infinite limits

HW #4 Due Fri. 9/04:
- Ch 1 review pp. 88-89 #3-67odd
- Test #1 Practice Problems handout
- 2.1 -#1-23odd — derivative definition

4. Find the limit (if it exists).

$$\lim_{x\to 0} \frac{5\sin 2x}{3x}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{5\cdot 2}{3}$$

$$= \left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right)\left(\lim_{x\to 0} \frac{10}{3}\right)$$

$$= 1 \cdot \frac{10}{3} = \boxed{\frac{10}{3}}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

If $x \to 0$,
then $2x \to 0$

$$\lim_{x\to 0} \frac{\sin 2x}{2x} = 1$$

5. Find the limit (if it exists).

$$\lim_{x \to -5} f(x), \quad f(x) = \begin{cases} -x^2 + 8, & x \le -5 \\ 2x + 3, & x > -5 \end{cases}$$

$-(-5)^2 + 8 = -25 + 8 = -17$
$2(-5) + 3 = -10 + 3 = -7$
non-removable (jump) discontinuity @ -5

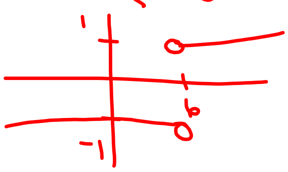
$\lim_{x \to -5} f(x)$ $\boxed{\text{does not exist}}$

$\lim_{x \to -5^-} f(x) = -17 \qquad \lim_{x \to -5^+} f(x) = -7$

f is continuous on $(-\infty, -5] \cup (-5, \infty)$

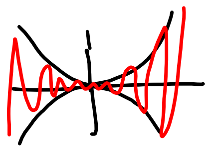
$$\lim_{x \to 6^-} \frac{|x-6|}{x-6} = \boxed{-1}$$

$$\frac{|x-6|}{x-6} = \begin{cases} \frac{x-6}{x-6} = 1, & x - 6 > 0, \; x > 6 \\ \frac{-(x-6)}{x-6} = -1, & x - 6 < 0, \; x < 6 \end{cases} = \begin{cases} 1, & x > 6 \\ -1, & x < 6 \end{cases}$$



7. Use the Squeeze Theorem to find $\lim\limits_{x\to 0} f(x)$.

$f(x) = 5x^2 \sin\frac{1}{x}$



$$-1 \le \sin\frac{1}{x} \le 1$$

$$-5x^2 \le 5x^2 \sin\frac{1}{x} \le 5x^2$$

$$\lim_{x\to 0}(-5x^2) \le \lim_{x\to 0} 5x^2 \sin\frac{1}{x} \le \lim_{x\to 0} 5x^2$$

$$0 \le \lim_{x\to 0} 5x^2 \sin\frac{1}{x} \le 0$$

$$\Rightarrow \lim_{x\to 0} 5x^2 \sin\frac{1}{x} = \boxed{0}$$

8. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2, 3]$ such that $f(c) = -4$, and if so, find all such values of c.

$f(x) = x^2 - 7x + 2$

1. evaluate $f(x)$ @ endpoints of given interval

$f(-2) = (-2)^2 - 7(-2) + 2 = 4 + 14 + 2 = 20$

$f(3) = 3^2 - 7(3) + 2 = 9 - 21 + 2 = -10$

2. compare to given function value

$\left.\begin{array}{l} 20 > -4 \\ -10 < -4 \end{array}\right\}$ IVT applies, i.e. there is a $c \in [-2, 3]$ such that $f(c) = -4$

3. Set $f(x) = f(c)$ & solve for x.

$x^2 - 7x + 2 = -4$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$\cancel{x = 6}$, $\boxed{x = 1}$

4. Throw out any solutions not in given interval

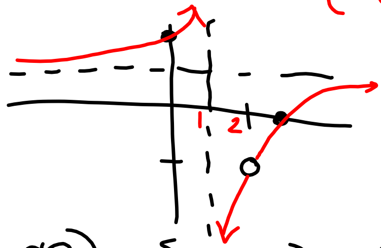
9. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-2)(x-5)}{(x-1)(x-2)}$$

non-removable discontinuity @ $x = 1$

removable discontinuity @ $x = 2$

f is continuous on $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$



$$(-\infty, \infty) - \{1, 2, 3\} = (-\infty, 1) \cup (1, 2) \cup (2, 3) \cup (3, \infty)$$

10. Find the limit (if it exists). $(a-b)(a+b) = a^2 - b^2$
$a^2 + ab - ab - b^2$

$$\lim_{x \to -2^+} \frac{\sqrt{x + 11} - 3}{x^2 + 5x + 6} = \lim_{x \to -2^+} \frac{\sqrt{x+11} - 3}{(x+2)(x+3)} \cdot \frac{\sqrt{x+11} + 3}{\sqrt{x+11} + 3}$$

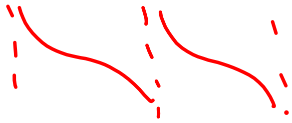
$$= \lim_{x \to -2^+} \frac{x + 11 - 9}{(x+2)(x+3)(\sqrt{x+11} + 3)} = \lim_{x \to -2^+} \frac{\cancel{x+2}\ 1}{\cancel{(x+2)}(x+3)(\sqrt{x+11} + 3)}$$

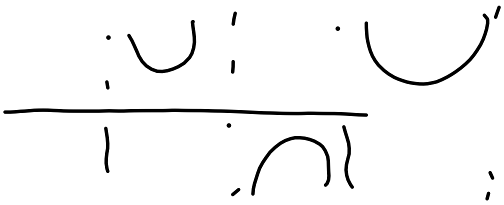
$$= \frac{1}{(-2+3)(\sqrt{-2+11} + 3)} = \frac{1}{1(\sqrt{9} + 3)} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

$$\lim_{x\to 5^+} \tan\left(\frac{\pi x}{2}\right)$$

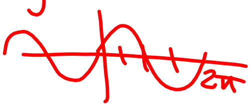
$$= \boxed{-\infty}$$

cot

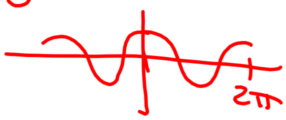




$y = \sin x$



$y = \cos x$



$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$ $\tan x = \frac{\sin x}{\cos x}$

5. Find the limit L, then use the $\varepsilon-\delta$ definition to prove that the limit is L.

$\cancel{\lim_{x \to 2}(2x+3)}$

$\lim_{x \to -4}(7-3x)$ $c = -4$ $\boxed{L=19}$

$= 7-3(-4)$

$= 7+12 = 19$ $\delta = \varepsilon/3$

$|f(x)-L| = |7-3x-19| = |-3x-12| = |-3(x+4)| =$

$= 3|x-(-4)| < \varepsilon \Rightarrow |x-(-4)| < \varepsilon/3$

Given $\varepsilon > 0$, there is a $\delta > 0$ such that $|x-c| < \delta$ implies $|f(x)-L| < \varepsilon$.
If $\delta = \varepsilon/3$, then whenever $|x-(-4)| < \delta$, we have that

$|f(x)-L| = |7-3x-19| = 3|x-(-4)| < 3\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon$

i.e. $|f(x)-L| < \varepsilon$.

Hence $\lim_{x \to -4}(7-3x)$ is indeed 19. $\square$