Homework for Test #2 on Derivatives

- 2.1 #1-23 odd Find the derivative by the limit process
- 2.1 #29-32 all find the equation of the tangent line
- 2.1 #61-69 odd Use the alternate form to find the derivative
- 2.1 #71-79 odd Describe x-values where the function is differentiable (given graph)
- 2.2 #3-51 odd Find the derivative using the basic derivative rules
- 2.2 #91-94 all; 101, 102 use the derivative to solve rate of change word problem
- 2.3 #1-53 odd, 63-69 odd, Product and quotient rules
- 2.3 #75-81 all, 83-91 odd, 109-115 all
- 2.4 #7-33 odd, 47-81 odd Chain rule
- 5.1 #45-61, 71 Logarithmic functions
- 5.4 #39-57 Exponential functions
- 5.5 #41-55 Log and exp functions with other bases
- 5.8 #41-59 Inverse trig functions

Fri.

QUIZ FRIDAY on derivative rules

Recommended:
Work through intuitive exercises on Khan Academy:

- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$
2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$
3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$
4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.2

22. $y = 5 + \sin x$

$y' = 0 + \cos x = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \frac{5}{8}\left(-3x^{-3-1}\right) + 2(-\sin x)$

$= \boxed{\frac{-15}{8}x^{-4} - 2\sin x}$

$= \frac{-15}{8x^4} - 2\sin x$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = 4x + 0 - x^{-2} = \boxed{4x - x^{-2}} = 4x - \frac{1}{x^2}$

$= \frac{4x^3 - 1}{x^2}$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = \boxed{36x - 45x^2}$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x$

$[x^n]' = nx^{n-1}$

$[\cos x]' = -\sin x$

$= 2x^{-1/3} + 3\cos x$

$f'(x) = \boxed{-\frac{2}{3}x^{-4/3} - 3\sin x}$

$= \frac{-2}{3\sqrt[3]{x^4}} - 3\sin x$

2.2 cont.

$s(t)$ = position — m

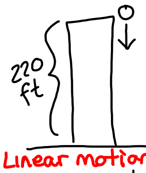
$v(t) = s'(t)$ = velocity — m/s

$a(t) = v'(t) = s''(t)$ = acceleration — $m/s/s = m/s^2$

average velocity: $\frac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92.



initial velocity $V_0 = -22$ ft/s

$v(3) = ?$

$v(t) = ?$ after falling 108 ft

Linear motion formula

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

$g = -9.8$ m/s²

$= -32$ ft/s²

position @ time t   acceleration   initial velocity $v(0)$ initial position $s(0)$

$s(t) = \frac{1}{2}(-32)t^2 + (-22)t + 220$

$s(t) = -16t^2 - 22t + 220$

$v(t) = s'(t) = -32t - 22$

$v(3) = -32(3) - 22 = -96 - 22 = \boxed{-118 \text{ ft/s}}$

$220 - 108 = -16t^2 - 22t + 220$

$16t^2 + 22t - 108 = 0$

$8t^2 + 11t - 54 = 0$

position after ball has fallen 108 ft from initial 220 ft

$t = \dfrac{-11 \pm \sqrt{11^2 - 4(8)(-54)}}{2(8)}$

$= 2 \leftarrow$ time it takes ball to fall 108 ft

$v(2) = -32(2) - 22 = -64 - 22 = \boxed{-86 \text{ ft/s}}$

The volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$

If the sphere is growing,

Find the rate of change of volume with respect to radius when the radius is 2 cm.

$V'(r) = \left(\frac{4}{3}\pi\right)(3r^2)$

$= 4\pi r^2$ = surface area of a sphere of radius r

$V'(2) = 4\pi(2)^2 = \boxed{16\pi \text{ cm}^2}$

Power Rule:
$\frac{d}{dx}[x^n] = nx^{n-1}$ d/dx [c]=0

Product Rule:
$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Constant Multiple Rule:
$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

Quotient Rule:
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

Sum & Difference:
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Chain Rule:
$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Trig Functions:

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.3 Product & Quotient Rules

$[fg]' = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

$\boxed{(fg)' = f'g + fg'}$

$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

"low dee high less high dee low,
draw the line and square below"

$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$

2.3

6. $g(x) = \sqrt{x}\sin x = (x^{1/2})(\sin x)$

$g'(x) = (x^{1/2})'(\sin x) + (x^{1/2})(\sin x)'$

$= \boxed{\left(\tfrac{1}{2}x^{-1/2}\right)(\sin x) + (x^{1/2})(\cos x)}$

$= \dfrac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$

12. $f(t) = \dfrac{\cos t}{t^3} = (\cos t)(t^{-3})$

$f'(t) = \dfrac{(t^3)(\cos t)' - (\cos t)(t^3)'}{(t^3)^2}$

$= \boxed{\dfrac{(t^3)(-\sin t) - (\cos t)(3t^2)}{(t^3)^2}}$

$= \dfrac{-t^3\sin t - 3t^2\cos t}{t^6}$

$= \dfrac{t^2(-t\sin t - 3\cos t)}{t^6} = \dfrac{-t\sin t - 3\cos t}{t^4}$

26. $f(x) = \dfrac{x^3+3x+2}{x^2-1}$

Note: as a product,
$f(x) = (x^3+3x+2)(x^2-1)^{-1}$
we don't know how to differentiate this yet
so we have to use the quotient rule!

$\cancel{f(x) = (x^3+3x+2)(x^2-1)^{-1}}$

$$f'(x) = \frac{(x^2-1)(x^3+3x+2)' - (x^3+3x+2)(x^2-1)'}{(x^2-1)^2}$$

$$= \boxed{\frac{(x^2-1)(3x^2+3) - (x^3+3x+2)(2x)}{(x^2-1)^2}}$$

Find the slope of the tangent line

$f(x) = 3x - \sin x \quad ; (\pi, 3\pi)$

$f'(x) = 3 - \cos x$

$m = f'(\pi) = 3 - \cos\pi = 3-(-1) = \boxed{4}$

slope of tangent line to f @ $(\pi, 3\pi)$

$y - 3\pi = 4(x-\pi)$

$y - 3\pi = 4x - 4\pi$

$y = 4x - \pi \leftarrow$ equation of the tangent line to f @ $(\pi, 3\pi)$

Find the equation of the tangent line.

$f(x)=2x^3+\sin x-2x$; $(0,0)$

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$f'(x) = 6x^2+\cos x-2$

$m=f'(0)=6(0)^2+\cos(0)-2 = 0+1-2 = -1$

$y-y_1=m(x-x_1)$

$y-0=-1(x-0)$

$\boxed{y=-x}$