Homework for Test #2 on Derivatives

HW #5 due Friday, 9/11

- 2.1 #1-23 odd Find the derivative by the limit process
- 2.1 #29-32 all find the equation of the tangent line
- 2.1 #61-69 odd Use the alternate form to find the derivative
- 2.1 #71-79 odd Describe x-values where the function is differentiable (given graph)
- 2.2 #3-51 odd Find the derivative using the basic derivative rules

HW #6 due Monday, 9/14

- 2.2 #91-94 all; 101, 102 use the derivative to solve rate of change word problem
- 2.3 #1-53 odd, 63-69 odd, Product and quotient rules

HW #7 due Wednesday, 9/16

- 2.3 #75-81 all, 83-91 odd, 109-115 all Rates of change, second derivatives
- 2.4 #7-33 odd, 47-81 odd Chain rule
- 5.1 #45-61, 71 Logarithmic functions
- 5.4 #39-57 Exponential functions
- 5.5 #41-55 Log and exp functions with other bases
- 5.8 #41-59 Inverse trig functions

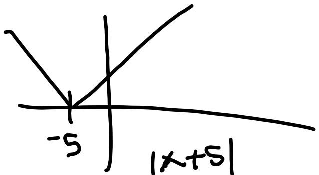
Recommended:
Work through intuitive exercises on Khan Academy:

- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$\lim_{x \to -2} \frac{x^3 + 2x^2 + 1 - \left((-2)^3 + 2(-2)^2 + 1\right)}{x - (-2)}$$

$$= f'(-2)$$



$$\lim_{x\to -5^-} \frac{\overset{|x+5|}{f(x)} - f(-5)}{x-(-5)} = \lim_{x\to -5^-} \frac{|x+5|}{x+5} = -1$$

$$\lim_{x\to -5^+} \frac{\overset{|x+5|}{f(x)} - f(-5)}{x-(-5)} = \lim_{x\to -5^+} \frac{|x+5|}{x+5} = +1$$

Power Rule:

$\frac{d}{dx}[x^n] = nx^{n-1}$ d/dx [c]=0

Constant Multiple Rule:

$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

Sum & Difference:

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Product Rule:

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule:

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

Chain Rule:

$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Trig Functions:

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.1

32. $f(x)=\frac{1}{x+1}$; $(0,1)$

Find the equation of the tangent line to the function at the given point.

$$\lim_{h\to 0}\frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{\frac{h}{1}}=\lim_{h\to 0}\frac{x+1-(x+h+1)}{(x+h+1)(x+1)}\cdot\frac{1}{h}=\lim_{h\to 0}\frac{-h}{(x+h+1)(x+1)\cdot h}$$

$$=\lim_{h\to 0}\frac{-1}{(x+h+1)(x+1)}=\frac{-1}{(x+1)^2}$$

$$m=\frac{-1}{(0+1)^2}=-1$$

$$y-1=-1(x-0)$$

$$\boxed{y=-x+1}$$

Find $f'(x)$

2.2

43. $f(x)=\frac{x^3-3x^2+4}{x^2}$

$$=\frac{x^3}{x^2}-\frac{3x^3}{x^2}+\frac{4}{x^2}=x-3+4x^{-2}$$

$$f'(x)=\boxed{1-8x^{-3}}$$

$$f'(x)=\frac{(x^2)(x^3-3x^2+4)'-(x^3-3x^2+4)(x^2)'}{(x^2)^2}$$

$$=\frac{x^2(3x^2-6x)-(x^3-3x^2+4)(2x)}{x^4}$$

$$=\frac{3x^4-\cancel{6x^3}-2x^4+\cancel{6x^3}-8x}{x^4}$$

$$=\frac{x^4-8x}{x^4}=\frac{x(x^3-8)}{x^4}=\frac{x^3-8}{x^3}=1-8x^{-3}$$

$$f(x)=\tan x=\frac{\sin x}{\cos x}$$

$$f'(x)=\frac{\cos x(\sin x)'-(\sin x)(\cos x)'}{(\cos x)^2}$$

$$=\frac{\cos x\cos x-(\sin x)(-\sin x)}{\cos^2 x}$$

$$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\boxed{\sec^2 x}$$

2.4 - The Chain Rule

$$[f(g[x])]' = \left[f'(g[x])\right] \cdot \left[g'[x]\right] \cdot [x]'$$

$$[f(g[h(x)])]' = f'(g[h(x)]) \cdot g'[h(x)] \cdot h'(x)$$

$$f(x) = \sin\left(x^5 - 3x^2\right)$$

$$f'(x) = \boxed{\left[\cos\left(x^5 - 3x^2\right)\right] \cdot \left(5x^4 - 6x\right)}$$

$$f(x) = \cos\left[5\tan(7x)\right]$$

$f(x) = \cos u$

$u = 5\tan v$

$v = 7x$

$$f'(x) = \left(-\sin\left[5\tan(7x)\right]\right)\cdot\left[5\sec^2(7x)\right]\cdot 7$$

$$f(x) = (5x)(\csc(x^2))$$

$$f'(x) = (5x)'(\csc(x^2)) + (5x)(\csc(x^2))'$$

$$= 5\csc(x^2) + 5x\left(-\csc(x^2)\cot(x^2)\right)\cdot(2x)$$

1. $f(x) = \cot(5x^2 - 3x)$

$f'(x) = \left[-\csc^2(5x^2-3x)\right] \cdot (10x - 3)$

$f(x) = 5\sin(3\cos 2x^5)$

$f'(x) = \left[5\cos(3\cos 2x^5)\right] \cdot (-3\sin(2x^5)) \cdot (10x^4)$

$$f(x) = (x)(\sec x)(\sqrt{x-1}) = (x \cdot \sec x) \cdot (x-1)^{1/2}$$

$$f'(x) = (x\sec x)'(x-1)^{1/2} + (x\sec x) \cdot \left[(x-1)^{1/2}\right]'$$

$$= \left[(x)'\sec x + x(\sec x)'\right](x-1)^{1/2} + (x\sec x) \cdot \tfrac{1}{2}(x-1)^{-1/2} \cdot (x-1)'$$

$$= \boxed{(\sec x + x\sec x\tan x)(x-1)^{1/2} + (x\sec x) \cdot \tfrac{1}{2}(x-1)^{-1/2} \cdot 1}$$