Homework for Test #2 on Derivatives

- 2.1 #1-23 odd Find the derivative by the limit process

HW #5 due Friday, 9/11

- 2.1 #29-32 all find the equation of the tangent line
- 2.1 #61-69 odd Use the alternate form to find the derivative
- 2.1 #71-79 odd Describe x-values where the function is differentiable (given graph)
- 2.2 #3-51 odd Find the derivative using the basic derivative rules

HW #6 due Monday, 9/14

- 2.2 #91-94 all; 101, 102 use the derivative to solve rate of change word problem
- 2.3 #1-53 odd, 63-69 odd, Product and quotient rules

HW #7 due Wednesday, 9/16

- 2.3 #75-81 all, 83-91 odd, 109-115 all Rates of change, second derivatives
- 2.4 #7-33 odd,

- 47-81 odd Chain rule
- 5.1 #45-61, 71 Logarithmic functions
- 5.4 #39-57 Exponential functions
- 5.5 #41-55 Log and exp functions with other bases
- 5.8 #41-59 Inverse trig functions

Recommended:
Work through intuitive exercises on Khan Academy:

- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$y = x\left(1 - \frac{4}{x+3}\right)$$

$$y = x - \frac{4x}{x+3}$$

$$y' = 1 - \frac{4(x+3) - 4x(1)}{(x+3)^2}$$

$y = \frac{4x^{3/2}}{x^{2/2}} = 4x^{1/2}$

$y' = 2x^{-1/2}$

Instantaneous rate of change of a function $f(x)$ when $x = c$ is $f'(c)$ <-- slope of tangent line through a single point

Average rate of change of a function $f(x)$ on the interval $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$ <-- slope of secant line through two points

Given a position function $s(t) = gt^2 + v_0 t + s_0$,

Since velocity is the rate of change of position,

The instantaneous velocity at time $t = c$ is $s'(c)$

The average velocity on the interval $[a, b]$ is $\frac{s(b)-s(a)}{b-a}$

Find $y', y'', y''', y^{(4)}, y^{(5)}, \dots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$ $y = x^6 + 2x^5 - 3x^4 + 2x - 5$

If $f(x)$ is a polynomial of degree n, then $f^{(n+1)}(x) = 0$.

If $f(x) = x^n$, then $f^{(n)}(x) = n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

$f(x) = 3x^9 - 15x^4 + 23x^{16} - 201x^7 - 3$ $f^{(17)} = 0$

Power Rule:

$\frac{d}{dx}[x^n] = nx^{n-1}$ d/dx [c]=0

Constant Multiple Rule:

$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

Sum & Difference:

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Product Rule:

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule:

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

Chain Rule:

$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Trig Functions:

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$$f(x) = \cos\left(\sqrt{\tan^2 x - 2x}\right) = \cos\left[(\tan^2 x - 2x)^{1/2}\right]$$

$$f'(x) = -\sin\left(\sqrt{\tan^2 x - 2x}\right) \cdot \frac{1}{2}(\tan^2 x - 2x)^{-1/2} \cdot \left[(\tan^2 x)' - (2x)'\right]$$

$$= -\sin\left(\sqrt{\tan^2 x - 2x}\right) \cdot \frac{1}{2}(\tan^2 x - 2x)^{-1/2} \cdot \left[\left[(\tan x)^2\right]' - 2\right]$$

$$= \boxed{-\sin\left(\sqrt{\tan^2 x - 2x}\right) \cdot \frac{1}{2}(\tan^2 x - 2x)^{-1/2} \cdot \left[2\tan x \cdot \sec^2 x - 2\right]}$$

$$\left[f(g(x))\right]' = f'(g(x)) \cdot g'(x)$$

$$\left(\left[g(x)\right]^2\right)' = 2 \cdot g(x) \cdot g'(x)$$

2. $f(x) = \sqrt[3]{\csc(4x)} = \left(\csc(4x)\right)^{1/3}$

$$f'(x) = \boxed{\frac{1}{3}\left(\csc(4x)\right)^{-2/3} \cdot \left(-\csc(4x)\cot(4x)\right) \cdot 4}$$

$$f(x) = \cot^2(\sin(3x)) = \left[\cot(\sin(3x))\right]^2$$

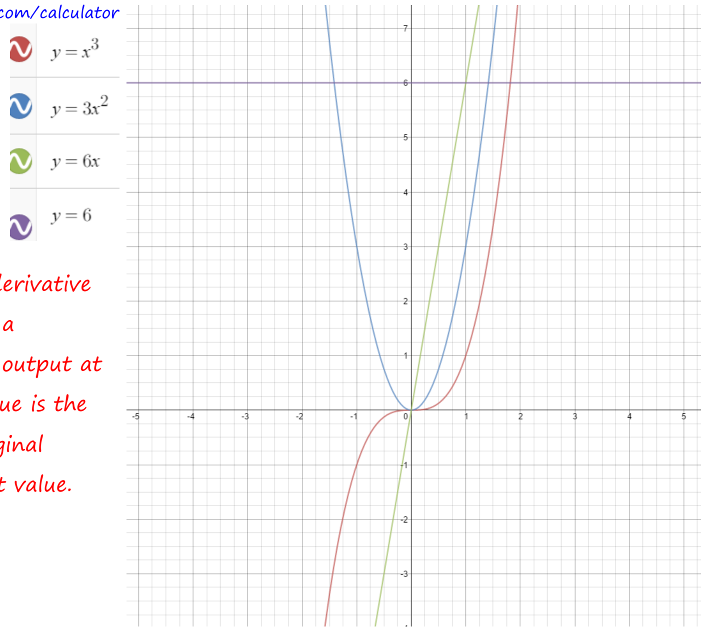
$$f'(x) = \boxed{2\cot(\sin(3x)) \cdot \left(-\csc^2(\sin(3x))\right) \cdot \cos(3x) \cdot 3}$$

3. $f(x) = \dfrac{\sin 2x}{x^3} = (\sin 2x)(x^{-3})$

$$f'(x) = (\sin 2x)'(x^{-3}) + (\sin 2x)(x^{-3})'$$

$$= \boxed{(2\cos 2x)(x^{-3}) + (\sin 2x)(-3x^{-4})}$$

https://www.desmos.com/calculator



Note that the derivative of a function is a function whose output at a particular value is the slope of the original function at that value.

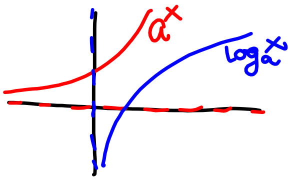
Ch 5 - Derivatives of Logarithmic and Exponential Functions

recall: $\ln x = \log_e x$

$e \approx 2.7$

$\log_2 8 = 3 \iff 2^3 = 8$

$\log_a b = c \iff a^c = b$



$y = 2^x$

$x =$ the power to which we raise 2 to get y

= the # of times we multiply 2 by itself to get y

$= \log_2 y$

$$\frac{d}{dx}\left[2^x\right] = 2^x \cdot \ln 2$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{x \cdot \ln 2}$$

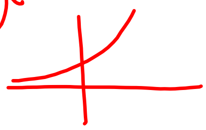
$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u \cdot \ln a}$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{f(x) \cdot \ln a} \cdot f'(x) = \frac{f'(x)}{f(x) \cdot \ln a}$$

$$\left[e^x\right]' = e^x \cdot \ln e = e^x \log_e e = e^x$$

$$\left[e^x\right]' = e^x$$



$$\left[\ln x\right]' = \frac{1}{x \ln e} = \frac{1}{x}$$

Since the derivative of e^x is itself, this means that graphically, at every x-value, the slope of the tangent line at that point is exactly the y-coordinate.

$$f(x) = \ln\left[\sin\left(5x^3+2x\right)\right]$$

$$f'(x) = \boxed{\frac{1}{\sin\left(5x^3+2x\right)} \cdot \cos\left(5x^3+2x\right) \cdot \left(15x^2+2\right)}$$

$$= \left(15x^2+2\right)\cot\left(5x^3+2x\right)$$

$$f(x) = (\sec x)\left(5^{\sin x}\right)$$

$$f'(x) = (\sec x)'\left(5^{\sin x}\right) + (\sec x)\left(5^{\sin x}\right)'$$

$$= \boxed{(\sec x\tan x)\cdot 5^{\sin x} + (\sec x)\cdot 5^{\sin x}\cdot \ln 5\cdot \cos x}$$