Homework for Test #2 on Derivatives

Test 2 - Wed. 9/23?

HW #5 due Friday, 9/11
- 2.1 #1-23 odd Find the derivative by the limit process
- 2.1 #29-32 all find the equation of the tangent line
- 2.1 #61-69 odd Use the alternate form to find the derivative
- 2.1 #71-79 odd Describe x-values where the function is differentiable (given graph)

HW #6 due Monday, 9/14
- 2.2 #3-51 odd Find the derivative using the basic derivative rules
- 2.2 #91-94 all; 101, 102 use the derivative to solve rate of change word problem

HW #7 due Wed. 9/16
- 2.3 #1-53 odd, 63-69 odd, Product and quotient rules
- 2.3 #75-81 all, 83-91 odd, 109-115 all Rates of change, second derivatives
- 2.4 #7-33 odd, 47-81 odd Chain rule

HW #8 due Tues. 9/22

HW #9 due Wed. 9/23
- 5.1 #45-61 odd, 71 Logarithmic functions
- 5.4 #39-57 odd Exponential functions
- 5.5 #41-55 odd Log and exp functions with other bases
- 5.8 #41-59 odd Inverse trig functions

Recommended:
Work through intuitive exercises on **Khan Academy**:
- Slope of secant lines
- Tangent lope is limiting value of secant slope
- Derivative intuition
- Visualizing derivatives
- Graphs of functions and their derivatives
- The formal and alternate form of the derivative
- Derivatives 1
- Recognizing slopes of curves
- Power rule
- Special derivatives

1. $f(x) = \cos(5x)$

$f'(x) = [-\sin(5x)] \cdot 5$

2. $f(x) = \ln(\tan x)$

$f'(x) = \frac{1}{\tan x} \cdot \sec^2 x$

3. $f(x) = e^{\arctan 2x}$

$f'(x) = e^{\arctan 2x} \cdot \frac{1}{1+(2x)^2} \cdot 2$

4. $f(x) = \sqrt[3]{\sec x} = (\sec x)^{1/3}$

$f'(x) = \frac{1}{3}(\sec x)^{-2/3} \cdot \sec x \tan x$

5. $f(x) = -4\sin(3e^x)$

$f'(x) = [-4\cos(3e^x)] \cdot 3e^x$

Find the derivative:

1. $f(x) = \csc x$

$f'(x) = -\csc x \cot x$

2. $s(t) = -16t^2 + 5t + 100$

$s'(t) = -32t + 5$

3. $y = \sqrt[3]{x} = x^{1/3}$

$y' = \frac{1}{3}x^{-2/3}$

4. $f(x) = \frac{5x^3 - 3x^2}{x} = 5x^2 - 3x$

$f'(x) = 10x - 3$

5. $f(x) = \cos(3x^2 - 4x)$

$f'(x) = [-\sin(3x^2 - 4x)](6x - 4)$

6. $f(x) = e^x$

$f'(x) = e^x$

7. $f(x) = \ln x$

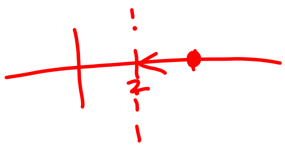
$f'(x) = \frac{1}{x}$

8. **Find $f^{(7)}(x)$** for $f(x) = 11x^6 - 12x^5 + 18x - 5$

$f^{(7)}(x) = 0$

Find the limit (if it exists).

9. $\lim\limits_{x \to 2^+} \frac{x-3}{x-2} = -\infty$



10. $\lim\limits_{x \to -1} \frac{x^2-1}{x+1} = \lim\limits_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}$

$= -1 - 1 = \boxed{-2}$

11. Apply the alternate definition of the derivative that deals with continuity, $f'(c) = \lim\limits_{x \to c} \frac{f(x)-f(c)}{x-c}$, to **explain why the derivative of $f(x) = |x+5|$ does not exist at $c = -5$.**

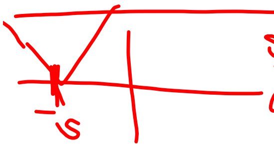
$\lim\limits_{x \to -5} \frac{|x+5| - |-5+5|}{x-(-5)} = \lim\limits_{x \to -5} \frac{|x+5|}{x+5}$

$\frac{|x+5|}{x+5} = \begin{cases} 1, & x > -5 \\ -1, & x < -5 \end{cases}$

$\lim\limits_{x \to -5^-} \frac{|x+5|}{x+5} = -1$

$\lim\limits_{x \to -5^+} \frac{|x+5|}{x+5} = 1$

Because the left- and right-hand limits are different, the limit in general and hence the derivative defined by that limit does not exist.



slope from left & right is different, sharp point. f' DNE

5.4 Find the equation of the tangent line to the graph of f at the indicated point.

78. $f(x) = \tan^2 x$; $\left(\frac{\pi}{4}, 1\right)$ $f(x) = [\tan x]^2$

$f'(x) = 2\tan x \cdot \sec^2 x$

$m = f'\left(\frac{\pi}{4}\right) = 2\tan\frac{\pi}{4} \cdot \left[\sec\frac{\pi}{4}\right]^2$

$= 2 \cdot 1 \cdot (\sqrt{2})^2 = 2 \cdot 1 \cdot 2 = 4$

$\boxed{y - 1 = 4\left(x - \frac{\pi}{4}\right)}$

$y - 1 = 4x - \pi$

$\boxed{y = 4x - \pi + 1}$

5.1

58. $f(x) = \ln\sqrt[3]{\frac{x-1}{x+1}} = \ln\left(\left[\frac{x-1}{x+1}\right]^{1/3}\right)$

$\log_a(b^p) = p\log_a b$

$\log_a \frac{M}{N} = \log_a M - \log_a N$

$= \frac{1}{3}\ln\frac{x-1}{x+1}$

$= \frac{1}{3}\left(\ln(x-1) - \ln(x+1)\right)$

$= \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$

$f'(x) = \boxed{\frac{1}{3(x-1)} - \frac{1}{3(x+1)}}$

5.8-ish

$f(x) = \arcsin(3x)$

$$f'(x) = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

$f(x) = \arctan(\ln(2x))$

$$f'(x) = \frac{1}{1+(\ln 2x)^2} \cdot \frac{1}{2x} \cdot 2$$

$f(x) = \cot\left(5^{\arcsin(4x^3)}\right)$

$f(x) = 5\sqrt{3\csc^2(\log_4(\text{arccsc}(7x^9)))}$

$$= 5\left[3\left(\csc\left[\log_4(\text{arccsc}[7x^9])\right]\right)^2\right]^{1/2}$$

$$= 5\sqrt{3}\csc\left[\log_4(\text{arccsc}[7x^9])\right]$$

$$\boxed{f'(x) = -5\sqrt{3}\csc(\log_4(\text{arccsc}(7x^9)))\cot(\log_4(\text{arccsc}[7x^9])) \cdot \frac{1}{(\ln 4)\,\text{arccsc}[7x^9]} \cdot \frac{-1}{|7x^9|\sqrt{(7x^9)^2-1}} \cdot 63x^8}$$

$$f(x) = \tan\left[\log_2\left(\sin\left[3^{5x}\right]\right)\right]$$

$$f'(x) = \sec^2\left[\log_2(\sin 3^{5x})\right] \cdot \frac{1}{\ln 2 \cdot \sin 3^{5x}} \cdot \cos 3^{5x} \cdot$$

$$\cdot\, 3^{5x} \ln 3 \cdot 5$$

$$f(x) = -2^{\ln(\arctan(7x^5 - \cos 3x))}$$

$$f'(x) = -2^{\ln(\arctan(7x^5 - \cos 3x))} \ln 2 \cdot$$

$$\cdot \frac{1}{\arctan(7x^5 - \cos 3x)} \cdot \frac{1}{1 + (7x^5 - \cos 3x)^2} \cdot (35x^4 + 3\sin 3x)$$