Homework for Test #3

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

What happens if...

$$x^2y + y^2x = -2$$

how to find y'?

2.5 Implicit Differentiation

☆ $y = f(x)$

y is a function of x

$\frac{d}{dx}[x] = 1$; $\frac{d}{dx}[y] = y'$

6. $x^2y + y^2x = -2$

$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}[-2]$

$(x^2y)' + (y^2x)' = 0$

$(x^2)'y + x^2y' + (y^2)'x + y^2x' = 0$

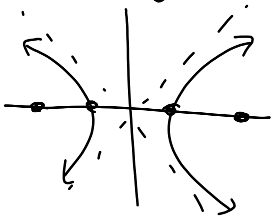
$2xy + x^2y' + 2y \cdot y'x + y^2 = 0$

$x^2y' + 2xyy' = -y^2 - 2xy$

$y'(x^2 + 2xy) = -y^2 - 2xy$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2 - y^2 = 16$



$$\frac{d}{dx}[x^2 - y^2] = \frac{d}{dx}[16]$$

$$2x - 2y \cdot y' = 0$$

$$-2yy' = -2x$$

$$y' = \frac{-2x}{-2y}$$

$$\boxed{y' = \frac{x}{y}}$$

8. $\sqrt{xy} = x - 2y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}[x - 2y]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \left[1 \cdot y + x \cdot y'\right] = 1 - 2y'$$

$$\frac{1}{2}(xy)^{-1/2} y + \frac{1}{2}(xy)^{-1/2} xy' = 1 - 2y'$$

$$2y' + \frac{1}{2}(xy)^{-1/2} xy' = 1 - \frac{1}{2}(xy)^{-1/2} y$$

$$y'\left(2 + \frac{1}{2}(xy)^{-1/2} x\right) = 1 - \frac{1}{2}(xy)^{-1/2} y$$

$$\boxed{y' = \frac{1 - \frac{1}{2}(xy)^{-1/2} y}{2 + \frac{1}{2}(xy)^{-1/2} x}}$$

8. $\sqrt{xy} = x - 2y$

$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x-2y\right]$

$\frac{1}{2}(xy)^{-1/2} \cdot \left[1 \cdot y + x \cdot y'\right] = 1 - 2y'$

$\frac{1}{2\sqrt{xy}}\left[y + xy'\right] = 1 - 2y'$

$\frac{y + xy'}{2\sqrt{xy}} = 1 - 2y'$

$y + xy' = 2\sqrt{xy}\,(1 - 2y')$

$y + xy' = 2\sqrt{xy} - 4y'\sqrt{xy}$

$xy' + 4y'\sqrt{xy} = 2\sqrt{xy} - y$

$y'(x + 4\sqrt{xy}) = 2\sqrt{xy} - y$

$\boxed{y' = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}}$

10. $2\sin x \cos y = 1$

$\frac{d}{dx}\left[(2\sin x)(\cos y)\right] = \frac{d}{dx}\left[1\right]$

$(2\cos x)\cos y + (2\sin x)(-\sin y) \cdot y' = 0$

$-2y'\sin x \sin y = -2\cos x \cos y$

$y' = \frac{-2\cos x \cos y}{-2\sin x \sin y}$

$\boxed{y' = \cot x \cot y}$

12. $(\sin\pi x + \cos\pi y)^2 = 2$

$$\frac{d}{dx}\left[(\sin\pi x + \cos\pi y)^2\right] = \frac{d}{dx}[2]$$

$$2(\sin\pi x + \cos\pi y)(\pi\cos\pi x - \pi y'\sin\pi y) = 0$$

$$(2\sin\pi x + 2\cos\pi y)(\pi\cos\pi x - \pi y'\sin\pi y) = 0$$

$$2\pi\sin\pi x\cos\pi x - 2\pi y'\sin\pi x\sin\pi y + 2\pi\cos\pi x\cos\pi y - 2\pi y'\sin\pi y\cos\pi y = 0$$

$$2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y = 2\pi y'\sin\pi x\sin\pi y + 2\pi y'\sin\pi y\cos\pi y$$

$$\frac{2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y}{2\pi\sin\pi x\sin\pi y + 2\pi\sin\pi y\cos\pi y} = y'$$

factor out & cancel 2π

$$\frac{\sin\pi x\cos\pi x + \cos\pi x\cos\pi y}{\sin\pi x\sin\pi y + \sin\pi y\cos\pi y} = y'$$

$$\frac{\cos\pi x(\sin\pi x + \cos\pi y)}{\sin\pi y(\sin\pi x + \cos\pi y)} = y'$$

$$\boxed{\frac{\cos\pi x}{\sin\pi y} = y'}$$

16. $x = \sec\frac{1}{y}$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec(y^{-1})\right]$$

$$1 = \sec(y^{-1})\tan(y^{-1})\cdot(-y^{-2})y'$$

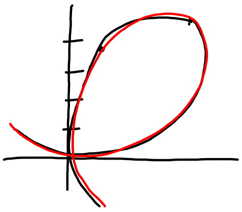
$$\frac{1}{\sec(y^{-1})\tan(y^{-1})(-y^{-2})} = y'$$

$$\boxed{-y^2\csc\frac{1}{y}\cot\frac{1}{y} = y'}$$

32. Folium of Descartes

$x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



$$\frac{d}{dx}\left[x^3+y^3-6xy\right]=0$$

$$3x^2+3y^2y'-6(1\cdot y+xy')=0$$

$$3x^2+3y^2y'-6y-6xy'=0$$

$$y'(3y^2-6x)=6y-3x^2$$

$$y'=\frac{3(2y-x^2)}{3(y^2-2x)} \qquad y'=\frac{2y-x^2}{y^2-2x}$$

$$m_{\left(\frac{4}{3},\frac{8}{3}\right)}=\frac{2\left(\frac{8}{3}\right)-\left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2-2\left(\frac{4}{3}\right)}=\frac{\frac{16}{3}-\frac{16}{9}}{\frac{64}{9}-\frac{8}{3}}\cdot\frac{9}{9}=\frac{16(3)-16}{64-8(3)}$$

$$=\frac{32}{40}=\boxed{\frac{4}{5}}$$