Homework for Test #3

2.5 # 1-39 odd; 43, 47 - Implicit Differentiation ← tues
2.6 # 15-23 odd - Related Rates ← wed
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

40. Find y'' in terms of x & y.

$y^2 = 4x$

$\frac{d^2y}{dy^2} = y''$ $\qquad$ $y' = \frac{dy}{dx} = \frac{d}{dx}[y]$

$\frac{d}{dx}[y^2] = \frac{d}{dx}[4x]$

$2y \cdot y' = 4$

$y' = \frac{4}{2y} = \frac{2}{y} = 2y^{-1}$

$\frac{d}{dx}[y'] = \frac{d}{dx}[2y^{-1}]$

$y'' = -2y^{-2} \cdot y'$

$y'' = -2y^{-2}(2y^{-1})$

$y'' = -4y^{-3}$

$y'' = \frac{-4}{y^3}$

2.6 Related Rates

18. $V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 2 \text{ in/min}$

$\frac{dV}{dt} = ?$ when $r = 6$ in

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \left(\frac{4}{3}\pi\right)\left(3r^2 \cdot \frac{dr}{dt}\right)$$

$$= 4\pi (6 \text{ in})^2 (2 \text{ in/min})$$

$$= 8\pi (36) \text{ in}^3\text{/min}$$

$$= \boxed{288\pi \text{ in}^3\text{/min}}$$

22. $V = \frac{1}{3}\pi r^2 h$

$\frac{dr}{dt} = 2 \text{ in/min}$

$h = 3r$

$\frac{dV}{dt} = ?$ when $r = 6$in

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right]$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[2r \cdot \frac{dr}{dt} \cdot h + r^2 \frac{dh}{dt}\right]$$

do not want

rewrite

$$V = \frac{1}{3}\pi r^2 (3r)$$

$$V = \pi r^3$$

$$\frac{dV}{dt} = 3\pi r^2 \cdot \frac{dr}{dt} = 3\pi (6 \text{ in})^2 \cdot (2 \text{ in/min})$$

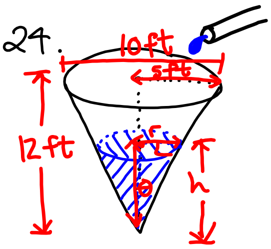
$$= \boxed{216\pi \text{ in}^3\text{/min}}$$

$8(36)$

$8(30+6)$

$= 240 + 48$

24.



$$\frac{r}{h} = \frac{5\text{ ft}}{12\text{ ft}}$$

$$r = \frac{5h}{12}$$

$$V = \frac{1}{3}\pi r^2 h$$

how to rewrite r in terms of h?

$$\frac{dV}{dt} = 10\ \frac{\text{ft}^3}{\text{min}}$$

$$\frac{dh}{dt} = ? \quad \text{when } h = 8\text{ ft}$$

$$V = \frac{1}{3}\pi\left(\frac{5h}{12}\right)^2 h$$

$$V = \frac{1}{3}\pi\frac{25h^2}{144}h$$

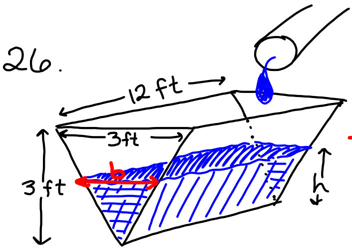
$$V = \frac{\pi}{3}\cdot\frac{25}{144}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{3}}\cdot\frac{25}{144}\cdot\cancel{3}h^2\cdot\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{25\pi}{144}h^2} = \frac{144(10)}{25\pi(8)^2}$$

$$= \frac{3\cdot\cancel{4}\cdot3\cdot\cancel{4}\cdot\cancel{2}\cdot\cancel{5}}{\cancel{5}\cdot5\cdot\pi\cdot2\cdot\cancel{4}\cdot\cancel{2}\cdot\cancel{4}} = \boxed{\frac{9}{10\pi}\ \frac{\text{ft}}{\text{min}}}$$

26.



$\frac{b}{h} = \frac{3}{3}$

$b = h$

$\frac{dV}{dt} = 2 \text{ ft}^3/\text{min}$

$\frac{dh}{dt} = ?$ when $h = 1$ ft

volume of a right prism is area of base times perpendicular height

$V_{trough} = (\text{area of } \nabla)(12 \text{ ft})$

$V = \left(\frac{1}{2}\, \text{base} \cdot h\right)(12 \text{ ft})$

$V = 6h^2$

$\frac{dV}{dt} = 12h \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{12h} = \frac{2}{12(1)}$

$= \boxed{\frac{1}{6} \text{ ft/min}}$

(b) If $\frac{dh}{dt} = \frac{3}{8}$ in/min when $h = 2$ ft find dV/dt

$\frac{dV}{dt} = (\cancel{12 \text{ ft}})(\cancel{2 \text{ ft}}) \cdot \left(\frac{3}{\cancel{8}_{4}} \cancel{\text{in}}/\text{min}\right) \cdot \frac{1 \text{ ft}}{\cancel{12 \text{ in}}}$

$= \boxed{\frac{3}{4} \text{ ft}^3/\text{min}}$