Homework for Test #3

HW #10 due Tues. 9/29:
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

$$\frac{d^2y}{dx^2} = y''$$

HW #11 due Fri. 10/2:
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Review

1. Use the alternate limit definition of the derivative to show that the function $f(x)=|x-3|+1$ is not differentiable at the point (3,1).

2. An object is thrown from a height of 12 feet with an initial velocity of -4 feet per second.

(a) Determine the velocity of the object after 1/2 of a second.

(b) Determine the average velocity of the object from 0 seconds to 3/4 of a second.

Review

1. *Use the alternate limit definition of the derivative to show that the function*

$(c, f(c))$

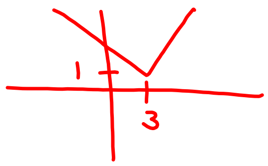
$f(x)=|x-3|+1$ is not differentiable at the point (3,1).

$$f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$$

$$f'(3) = \lim_{x \to 3} \frac{|x-3|+1-1}{x-3} = \lim_{x \to 3} \frac{|x-3|}{x-3}$$

$$\frac{|x-3|}{x-3} = \begin{cases} -1, & x<3 \\ 1, & x>3 \end{cases} \qquad \lim_{x \to 3^-} \frac{|x-3|}{x-3} = -1 \qquad \lim_{x \to 3^+} \frac{|x-3|}{x-3} = 1$$

Since the left- and right-hand limits are different, the limit in general, and hence the derivative defined by that limit, does not exist.



2. *An object is thrown from a height of* $\overset{s_0}{12}$ *feet with an initial velocity of* $\underset{v_0}{-4}$ *feet per second.* $s(t) = \frac{1}{2}at^2 + v_0 t + s_0$ $\qquad a = -32\ ft/s^2$

(a) *Determine the velocity of the object after 1/2 of a second.*
instantaneous rate of change of position

(b) *Determine the average velocity of the object from 0 seconds to 3/4 of a second.*
average rate of change of position

$$s(t) = -16t^2 - 4t + 12$$

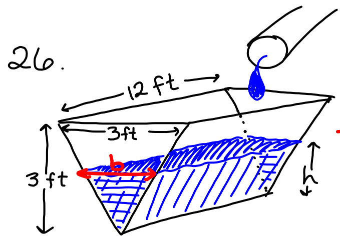
(a) $v(t) = s'(t) = -32t - 4$

$$v(\tfrac{1}{2}) = -32(\tfrac{1}{2}) - 4 = -16 - 4 = \boxed{-20\ ft/s}$$

(b) $$= \frac{s(\frac{3}{4}) - s(0)}{\frac{3}{4} - 0} = \frac{-16(\frac{3}{4})^2 - 4(\frac{3}{4}) + 12 - 12}{\frac{3}{4}}$$

$$= \frac{-\cancel{16}(\frac{9}{\cancel{16}}) - 3}{\frac{3}{4}} = -12 \cdot \frac{4}{3} = \boxed{-16\ ft/s}$$

26.



$\frac{b}{h} = \frac{3}{3}$

$b = h$

$\frac{dV}{dt} = 2 \text{ ft}^3/\text{min}$

$\frac{dh}{dt} = ?$ when $h = 1$ ft

volume of a right prism is base times perpendicular height

$V_{trough} = (\text{area of } \triangledown)(12 \text{ ft})$

$V = \left(\frac{1}{2}\,\underline{\text{base}} \cdot h\right)(12 \text{ ft})$

$V = 6h^2$

$\frac{dV}{dt} = 12h \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{12h} = \frac{2}{12(1)}$

$= \boxed{\frac{1}{6} \text{ ft/min}}$

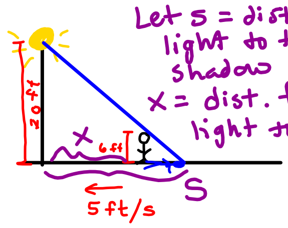
(b) $\frac{dh}{dt} = 3/8$ in/min

$\frac{dV}{dt} = ?$ when $h = 2$ ft

$\frac{dV}{dt} = (\cancel{12}\text{ft})(\cancel{2}\text{ ft})\left(\frac{3}{\cancel{8}_{4}} \text{ in/min}\right) \cdot \frac{1 \text{ ft}}{\cancel{12}\text{in}}$

$= \boxed{\frac{3}{4} \text{ ft}^3/\text{min}}$

36. A man 6 ft tall walks toward a light that is 20 ft above the ground at a rate of 5 ft/s. When he is 10 ft from the base of the light,
(a) at what rate is the tip of his shadow moving?



Let s = dist. from light to tip of shadow
x = dist. from light to man

$\frac{ds}{dt} = ?$ when $x = 10$ ft

$\frac{dx}{dt} = -5$ ft/s

$\frac{20}{s} = \frac{6}{s-x}$

$20(s-x) = 6s$

$20s - 20x = 6s$

$14s = 20x$

$7s = 10x$

$\frac{d}{dt}[7s] = \frac{d}{dt}[10x]$

$7 \cdot \frac{ds}{dt} = 10 \cdot \frac{dx}{dt}$

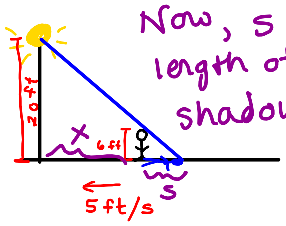
$\frac{ds}{dt} = \frac{10}{7} \cdot \frac{dx}{dt}$

$= \frac{10}{7}(-5)$

$= \boxed{-50/7 \text{ ft/s}}$

36. A man 6 ft tall walks toward a light that is 20 ft above the ground at a rate of 5 ft/s. When he is 10 ft from the base of the light,

(b) at what rate is the length of his shadow changing?



Now, s = length of shadow

$\frac{dx}{dt} = -5 \text{ ft/s}$

$\frac{ds}{dt} = ?$ when $x = 10$ ft

$$\frac{20}{x+s} = \frac{6}{s}$$

$$20s = 6(x+s)$$
$$20s = 6x + 6s$$
$$14s = 6x$$
$$7s = 3x$$

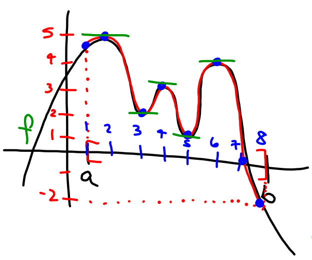
$$\frac{d}{dt}[7s] = \frac{d}{dt}[3x]$$

$$7 \cdot \frac{ds}{dt} = 3 \cdot \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{3}{7} \cdot \frac{dx}{dt}$$
$$= \frac{3}{7}(-5) = \boxed{-\frac{15}{7} \text{ ft/s}}$$

3.1 Extrema on an Interval

- maxima & minima
- relative & absolute



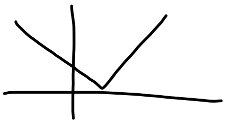
relative minima: (3,2), (5,1)

relative maxima: (2,5), (4,3), (6,4)

absolute maximum: 5 @ (2,5)

absolute minimum: -2 @ (8,-2)

$f(x)$ has a relative maximum or minimum when $f'(x) = 0$. or $f'(x)$ is undefined.



We call such x-values Critical #'s of f.

3.1 Find the absolute max & min on the closed interval.

28. $h(t) = \dfrac{t}{t-2}$, $[3,5]$

$h'(t) = \dfrac{(t-2)\cdot 1 - t(1)}{(t-2)^2} = \dfrac{-2}{(t-2)^2}$

* the only critical # (2) is not in $[3,5]$

$h(3) = \dfrac{3}{3-2} = \boxed{3} \leftarrow$ abs max

$h(5) = \dfrac{5}{5-2} = \boxed{\dfrac{5}{3}} \leftarrow$ abs min

30. $g(x) = \sec x$, $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

Find the absolute max & min on the closed interval.

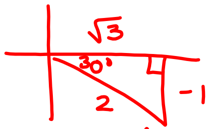
$g'(x) = \sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

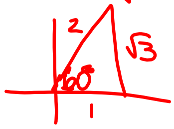
$\frac{\sin x}{\cos^2 x} = 0$ $x = 0$ is a critical #

$g(0) = \sec 0 = \boxed{1 \leftarrow \text{abs min}}$

$g\left(\frac{-\pi}{6}\right) = \sec\left(\frac{-\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$g\left(\frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = \boxed{2 \leftarrow \text{abs max}}$





$1 < 3 < 4$

$1 < \sqrt{3} < 2$

$1 > \frac{1}{\sqrt{3}} > \frac{1}{2}$

$2 > \frac{2}{\sqrt{3}} > 1$