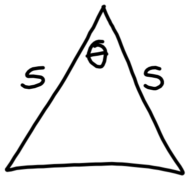
Given a function $f(x)$ that is continuous at a point $(c, f(c))$, explain in as many different ways as possible (in clear sentences), the meaning of $f'(c)$ and what is necessary for $f'(c)$ to be defined.

<u>Homework for Test #3</u>

HW #10 due Tues. 9/29:
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

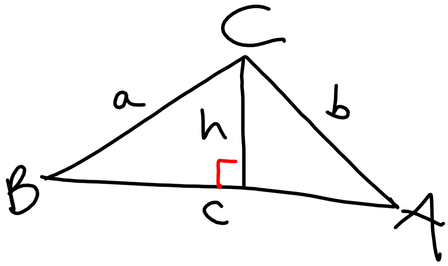
HW #11 due Fri. 10/2:
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity



$$A = \frac{1}{2}s^2 \sin\theta$$

$$\frac{dA}{dt} = \frac{1}{2}s^2 \cos\theta \cdot \frac{d\theta}{dt}$$



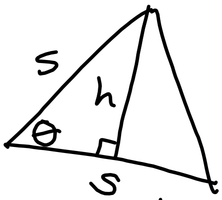
$$\sin B = \frac{h}{a}$$

$$\sin A = \frac{h}{b}$$

$$h = a\sin B = b\sin A$$

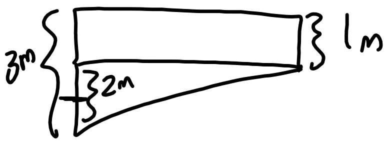
$$\frac{1}{2}c \cdot$$

$$\text{area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}bc\sin A$$
$$= \frac{1}{2}ac\sin B$$



$$\sin\theta = \frac{h}{s} \Rightarrow h = \sin\theta$$

$$\text{area} = \frac{1}{2}\, s \cdot s \cdot \sin\theta$$



$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

30. $g(x) = \sec x \quad , \left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$

Find the absolute max & min on the closed interval.

$g'(x) = \sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \frac{\sin x}{\cos^2 x}$

$\sin x = 0$ $\quad$ $\cos x = 0$

$x = 0$ $\quad$ no x's in $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$

critical # : 0

$\sec 0 = 1$

$\sec \frac{-\pi}{6} = 2/\sqrt{3}$

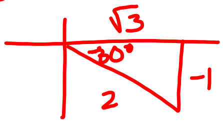
$\sec \frac{\pi}{3} = 2$

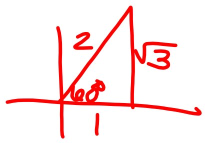
$1 < 3 < 4$

$1 < \sqrt{3} < 2$

$1 > \frac{1}{\sqrt{3}} > \frac{1}{2}$

$2 > \frac{2}{\sqrt{3}} > 1$





absolute minimum value is 1 & it occurs when $x = 0$

absolute maximum value is 2 & it occurs when $x = \frac{\pi}{3}$

22. $f(x) = x^3 - 12x \quad , [0, 4]$

Find the absolute max & min on the closed interval.

$f'(x) = 3x^2 - 12$ $\quad$ * The only critical # in $[0,4]$ is 2

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$f(2) = 2^3 - 12(2) = -16$

$f(0) = 0$

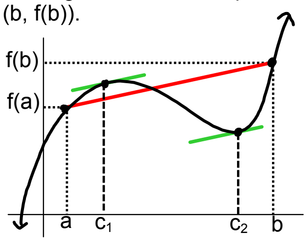
$f(4) = 4^3 - 12(4) = 16$

The absolute minimum is -16 and it occurs when $x = 2$ (@ $(2, -16)$)

The absolute maximum is 16 and it occurs when $x = 4$ @ $(4, 16)$

3.2 Rolle's Theorem & The Mean Value Theorem

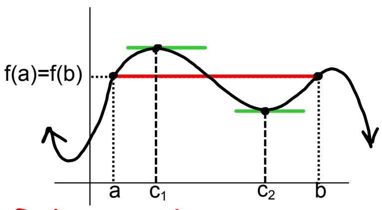
The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



If $f(x)$ is continuous on $[a,b]$ & differentiable on (a,b), then there exists $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



If $f(a)=f(b)$,

$$\frac{f(b)-f(a)}{b-a} = 0$$

If f is continuous on $[a,b]$ & differentiable on (a,b) AND $f(a)=f(b)$ then there exist $c \in (a,b)$ such that $f'(c)=0$.

Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}\ , \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|\ , \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4\ ,\ [1,4]$

f is continuous on [1,4] & differentiable on (1,4) ✓

Is $f(b) = f(a)$? yes

$f(1) = 1 - 5 + 4 = 0$

$f(4) = 4^2 - 5(4) + 4 = 0$

Rolle's Theorem applies

$f'(x) = 2x - 5$

$2x - 5 = 0$

$\boxed{x = 5/2}$

Can the Mean Value Theorem be applied?
If so, find all guaranteed values of c in (a,b).

34. $f(x) = \frac{x+1}{x}$, $\left[\frac{1}{2}, 2\right]$

$f'(x) = \frac{x(1)-(x+1)\cdot 1}{x^2} = -\frac{1}{x^2}$

Steps to solve MVT problems:
1. Is f continuous on [a,b]? yes
2. Is f differentiable on (a,b)? yes } MVT applies
3. Find (f(b)-f(a))/(b-a) $= \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{2-\frac{1}{2}} = \frac{\frac{3}{2}-\frac{6}{2}}{3/2} = -1$
4. Find f'(x)
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$\boxed{x=1}$

$\frac{-1}{x^2} = -1$

$-1 = -x^2$

$1 = x^2$

$\pm 1 = x$

38. $f(x) = 2\sin x + \sin 2x$, $[0, \pi]$

Is $f(x)$ cts. on $[0,\pi]$? yes
Is $f(x)$ diff. on $(0,\pi)$? yes } Mean Value Theorem applies

$\frac{f(b)-f(a)}{b-a} = \frac{2\sin\pi + \sin 2\pi - (2\sin 0 + \sin 2(0))}{\pi - 0}$

$= \frac{0+0-0-0}{\pi} = 0$

$f'(x) = 2\cos x + 2\cos 2x$

$2\cos x + 2\cos 2x = 0$

$2(\cos x + \cos 2x) = 0$

$\cos x + \cos 2x = 0$

$\cos x + 2\cos^2 x - 1 = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\qquad$ $\cos x + 1 = 0$

$\cos x = 1/2$ $\qquad$ $\cos x = -1$

$\boxed{x = \frac{\pi}{3}}$ $\qquad$ $\cancel{x = \pi} \notin (0,\pi)$