HW #10 due Tues. 9/29:
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

HW #11 due Fri. 10/2:
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

HW #12 due Mon. 10/5
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem

HW #11 due Wed. 10/7
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Test #3 - Friday/Monday?

3.1

31. $y = \frac{4}{x} + \tan\left(\frac{\pi x}{8}\right)$, $[1,2]$

$= 4x^{-1} + \tan\frac{\pi x}{8}$

$y' = -4x^{-2} + \sec^2\frac{\pi x}{8} \cdot \frac{\pi}{8}$

$0 = \frac{-4}{x^2} + \frac{\pi}{8}\sec^2\frac{\pi x}{8}$

$\frac{\pi}{8}\sec^2\frac{\pi x}{8} = \frac{4}{x^2}$

Solve$\left(\frac{-4}{x^2} + \frac{\pi}{8}\sec^2\frac{\pi x}{8} = 0, x\right)$

$\tan\frac{\pi}{8} = \tan\frac{\pi/4}{2}$

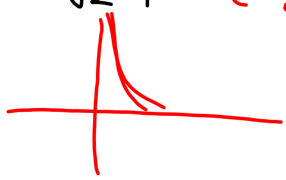
$= \frac{1 - \cos\pi/4}{\sin\pi/4} = \frac{1 - \sqrt{2}/2}{1/\sqrt{2}}$

$= \frac{2-\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{1}$

$= \sqrt{2} - 1$

$(2, 3)$

$(1, \sqrt{2} + 3)$



3.2

13. $f(x) = \dfrac{x^2-2x-3}{x+2}$, $[-1, 3]$

$f(3) \neq f(-1) \Rightarrow$ Rolle's Thm does not apply

3.2

17. $f(x) = \dfrac{6x}{\pi} - 4\sin^2 x$, $[0, \frac{\pi}{6}]$

$f'(x) = \dfrac{6}{\pi} - 8\sin x \cos x$

$\dfrac{6}{\pi} - 8\sin x \cos x = 0$

$-8\sin x \cos x = \dfrac{-6}{\pi}$

$2\sin x \cos x = \dfrac{3}{2\pi}$

$\sin 2x = \dfrac{3}{2\pi}$

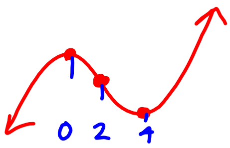
$2x = \sin^{-1}\left(\dfrac{3}{2\pi}\right) + 2\pi k$

$x = \dfrac{\sin^{-1}\left(\frac{3}{2\pi}\right) + 2\pi k}{2} = \boxed{0.249}$

- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

16. $f(x) = x^3 - 6x^2 + 15$



$f'(x) = 3x^2 - 12x$

$3x(x-4) = 0$

critical #'s: 0, 4

$f'(-1)$, $f'(1)$, $f'(5)$

$+ \; 0 \; - \; 4 \; +$

f is increasing on $(-\infty, 0) \cup (4, \infty)$
f is decreasing on $(0, 4)$
f has a relative maximum @ $(0, 15)$
f has a relative minimum @ $(4, -17)$
f has no absolute extrema, as its range is $(-\infty, \infty)$

$f''(x) = 6x - 12$

$6(x-2) = 0$

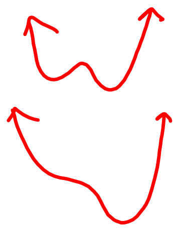
$f''(0)$, $f''(3)$

$- \; 2 \; +$

f is concave down on $(-\infty, 2)$
f is concave up on $(2, \infty)$
f has an inflection point @ $(2, -1)$

3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$



$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

critical #'s: 0, 3

$f'(-1)$, $f'(1)$, $f'(4)$

$- \; 0 \; - \; 3 \; +$

f is decreasing on $(-\infty, 3)$
f is increasing on $(3, \infty)$
f has a relative/absolute minimum @ $(3, -27)$

$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$x = 0, x = 2$

$f''(-1)$, $f''(1)$, $f''(3)$

$+ \; 0 \; - \; 2 \; +$

f is concave up on $(-\infty, 0) \cup (2, \infty)$
f is concave down on $(0, 2)$
f has inflection points @ $(0, 0)$ & $(2, -16)$

3.3

30. $f(x)=\dfrac{x+3}{x^2}$

$f'(x)=\dfrac{x^2(1)-(x+3)(2x)}{(x^2)^2}=\dfrac{x^2-2x^2-6x}{x^4}=\dfrac{-x^2-6x}{x^4}=\dfrac{-x(x+6)}{x^4}$

$0=\dfrac{-(x+6)}{x^3}$

critical #'s: $-6, 0$

$f'(-7)$, $f'(-1)$, $f'(1)$

$-$ -6 $+$ 0 $-$

f is decreasing on $(-\infty,-6)\cup(0,\infty)$
f is increasing on $(-6,0)$
f has a relative minimum @ $(-6,-1/12)$

$f''(x)=\dfrac{x^3(-1)+(x+6)(3x^2)}{(x^3)^2}=\dfrac{-x^3+3x^3+18x^2}{x^6}=\dfrac{2x^3+18x^2}{x^6}$

$=\dfrac{2x^2(x+9)}{x^6}$

$\dfrac{2(x+9)}{x^4}=0$

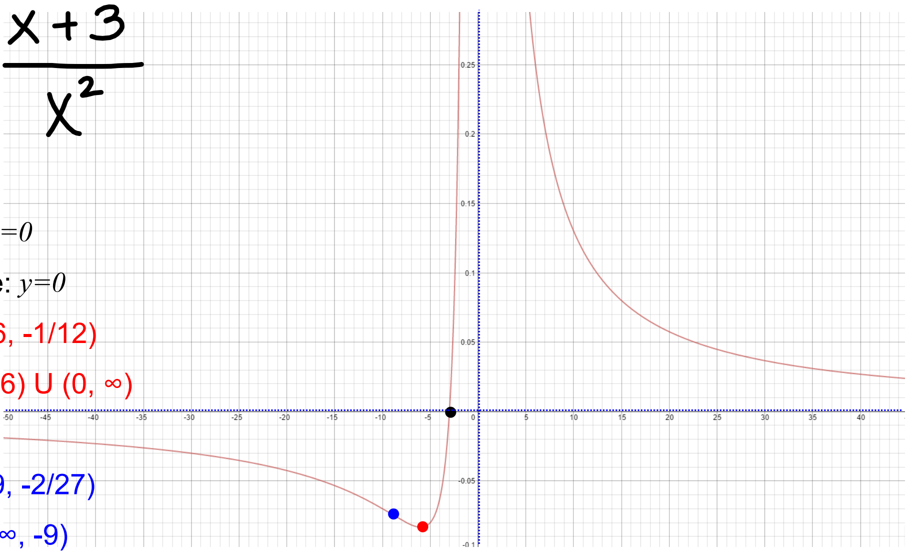
$x=0,-9$

$f''(-10)$, $f''(-1)$, $f''(1)$

$-$ -9 $+$ 0 $+$

f is concave down on $(-\infty,-9)$
f is concave up on $(-9,0)\cup(0,\infty)$
f has an inflection point @ $(-9,-2/27)$

30. $f(x)=\dfrac{x+3}{x^2}$



f:
- x-intercept: (-3,0)
- y-intercept: none
- vertical asymptote: $x=0$
- horizontal asymptote: $y=0$

f':
- relative minimum: (-6, -1/12)
- decreasing on: (-∞, -6) U (0, ∞)
- increasing on: (-6,0)

f'':
- inflection point at: (-9, -2/27)
- concave down on: (-∞, -9)
- concave up on (-9,0) U (0, ∞)