HW #10 due Tues. 9/29:
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

HW #11 due Fri. 10/2:
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

HW #12 due Mon. 10/5
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem

HW #13 due Wed. 10/7
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Test #3

Part A Friday, 10/09
2.5-2.6
Implicit Differentiation & Related Rates

Part B Monday, 10/12
3.1-3.4
Extrema, Concavity, Mean Value Theorem

3.3

30. $f(x) = \frac{x+3}{x^2}$

$f'(x) = \frac{x^2(1)-(x+3)(2x)}{(x^2)^2} = \frac{x^2-2x^2-6x}{x^4} = \frac{-x^2-6x}{x^4} = \frac{-x(x+6)}{x^4}$

$0 = \frac{-(x+6)}{x^3}$

critical #'s: $-6, 0$

$f'(-7)$, $f'(-1)$, $f'(1)$

$-$ -6 $+$ 0 $-$

f is decreasing on $(-\infty,-6)\cup(0,\infty)$
f is increasing on $(-6,0)$
f has a relative minimum @ $(-6, -1/12)$

$f''(x) = \frac{x^3(-1)+(x+6)(3x^2)}{(x^3)^2} = \frac{-x^3+3x^3+18x^2}{x^6} = \frac{2x^3+18x^2}{x^6}$

$= \frac{2x^2(x+9)}{x^6}$

$f''(-10)$, $f''(-1)$, $f''(1)$

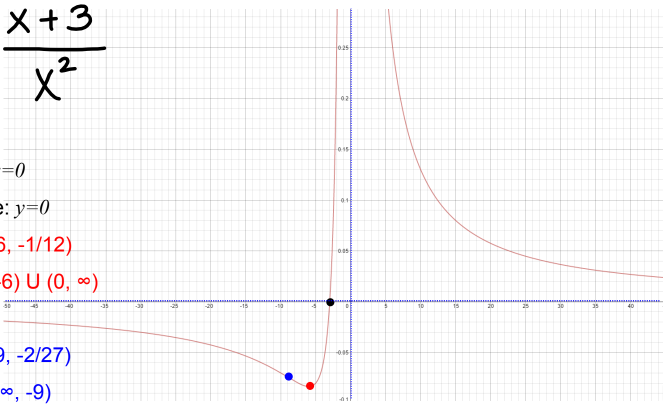
$-$ -9 $+$ 0 $+$

$\frac{2(x+9)}{x^4} = 0$

$x = 0, -9$

f is concave down on $(-\infty,-9)$
f is concave up on $(-9,0)\cup(0,\infty)$
f has an inflection point @ $(-9, -2/27)$

30. $f(x) = \dfrac{x+3}{x^2}$



f:
- x-intercept: (-3,0)
- y-intercept: none
- vertical asymptote: $x=0$
- horizontal asymptote: $y=0$

f':
- relative minimum: (-6, -1/12)
- decreasing on: (-∞, -6) U (0, ∞)
- increasing on: (-6,0)

f'':
- inflection point at: (-9, -2/27)
- concave down on: (-∞, -9)
- concave up on (-9,0) U (0, ∞)

3.4 #20

$f(x) = \dfrac{x+1}{\sqrt{x}}$

domain: $(0, \infty)$

$f'(x) = \dfrac{\sqrt{x}(1) - (x+1)\cdot\frac{1}{2\sqrt{x}}}{x} = \dfrac{\sqrt{x} - \frac{x+1}{2\sqrt{x}}}{x} = \dfrac{2x-(x+1)}{2x\sqrt{x}} = \dfrac{x-1}{2x^{3/2}}$

critical #'s: 0, 1

$f'(0.5)$ $f'(2)$

0 — 1 +

$\boxed{\text{f is decreasing on } (0,1) \\ \text{f is increasing on } (1,\infty) \\ \text{f has a relative min @ } (1,2)}$

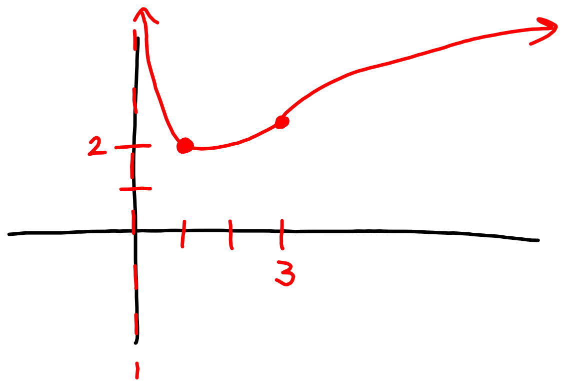
$f''(x) = \dfrac{2x^{3/2}(1) - (x-1)(3x^{1/2})}{4x^3} = \dfrac{2x^{3/2} - 3x^{3/2} + 3x^{1/2}}{4x^3}$

$= \dfrac{3x^{1/2} - x^{3/2}}{4x^3} = \dfrac{x^{1/2}(3-x)}{4x^3} = \dfrac{3-x}{4x^{5/2}}$

$\boxed{\text{f has an inflection point @ } (3, 4/\sqrt{3}) \\ \text{f is concave up on } (0,3) \\ \text{f is concave down on } (3,\infty)}$

$f''(1)$ $f''(4)$

0 + 3 —



Find y' implicitly in terms of x and y. $y' = \frac{dy}{dx} = \frac{d}{dx}[y]$ Review

$x^2y + 3xy^3 = 5x^3y^2$

$$\frac{d}{dx}\left[x^2y + 3xy^3\right] = \frac{d}{dx}\left[5x^3y^2\right]$$

$$(2xy + x^2y') + (3y^3 + (3x)(3y^2y')) = 15x^2y^2 + (5x^3)(2yy')$$

$$x^2y' + 9xy^2y' - 10x^3yy' = 15x^2y^2 - 2xy - 3y^3$$

$$y'(x^2 + 9xy^2 - 10x^3y) = 15x^2y^2 - 2xy - 3y^3$$

$$\boxed{y' = \frac{15x^2y^2 - 2xy - 3y^3}{x^2 + 9xy^2 - 10x^3y}}$$

$\cos x + \sin y = \tan(xy)$

$\frac{d}{dx}[\cos x + \sin y] = \frac{d}{dx}[\tan(xy)]$

$-\sin x + y'\cos y = [\sec^2(xy)](1\cdot y + x\cdot y')$

$-\sin x + y'\cos y = y\sec^2(xy) + xy'\sec^2(xy)$

$y'\cos y - xy'\sec^2(xy) = y\sec^2(xy) + \sin x$

$\boxed{y' = \frac{y\sec^2(xy) + \sin x}{\cos y - x\sec^2(xy)}}$

4. $x^3 + y^3 - 6xy = 0$

$\frac{d}{dx}[x^3 + y^3 - 6xy] = \frac{d}{dx}[0]$

$3x^2 + 3y^2y' - 6(1\cdot y + xy') = 0$

$3x^2 + 3y^2y' - 6y - 6xy' = 0$

$3y^2y' - 6xy' = 6y - 3x^2$

$y'(3y^2 - 6x) = 6y - 3x^2$

$y' = \frac{3(2y - x^2)}{3(y^2 - 2x)} = \boxed{\frac{2y - x^2}{y^2 - 2x}}$

1. Find the average rate of change of the volume of a sphere with respect to radius, as the radius of the sphere changes from 1 cm to 2 cm.

$V = \frac{4}{3}\pi r^3$

average rate of change of f from a to b

$\frac{f(b) - f(a)}{b-a}$

$\frac{V(2) - V(1)}{2-1}$

$= \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{1} = \boxed{\frac{28\pi}{3}\text{ cm}^2}$

2. Find the instantaneous rate of change of the volume of a sphere with respect to radius when the radius is 2 cm.

$V = \frac{4}{3}\pi r^3$

instantaneous rate of change of f when $x=c$ is $f'(c)$

$\frac{dV}{dr} = 4\pi r^2$

$\left.\frac{dV}{dr}\right|_{r=2\text{cm}} = 4\pi(2\text{ cm})^2 = \boxed{16\pi\text{ cm}^2}$

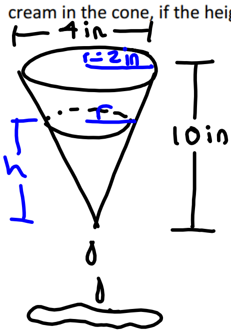
3. If the radius of a sphere changes at a rate of 3cm per second, find the rate of change of the volume of the sphere <u>with respect to time</u> when the radius is 2 cm.

$V = \frac{4}{3}\pi r^3$; $\frac{dV}{dt} = ?$ when $r = 2cm$ & $\frac{dr}{dt} = 3 cm/s$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (2 cm)^2 \cdot (3 cm/s) = \boxed{48\pi \ cm^3/s}$

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = ?$ when $h = 5in$

$V = \frac{1}{3}\pi \left(\frac{h}{5}\right)^2 \cdot h$

$\frac{dh}{dt} = \frac{1 in}{5 min}$

$V = \frac{\pi}{75} h^3$

$\frac{r}{h} = \frac{2}{10} \Rightarrow r = \frac{h}{5}$

$\frac{dV}{dt} = \frac{\pi}{25} h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{25} (5 in)^2 \cdot \frac{1 in}{5 min}$ -

$- \boxed{\frac{\pi}{5} \ in^3/min}$

12. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}t$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. Volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height.

$r = \sqrt{t+2}$

$h = \frac{1}{2}t$

$V = \pi r^2 h$

$\frac{dV}{dt} = ?$

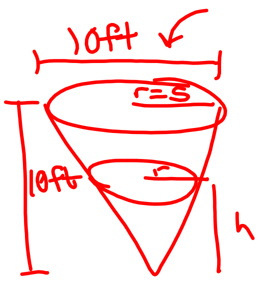
$$V = \pi\left(\sqrt{t+2}\right)^2 \cdot \frac{1}{2}t$$

$$= \pi(t+2)\cdot\frac{1}{2}t$$

$$V = \frac{\pi}{2}t^2 + \pi t$$

$$\frac{dV}{dt} = \boxed{\pi t + \pi}$$

13. A ~~conical tank~~ waffle cone is 10 feet across at the top and 10 feet deep. If it is being filled with ~~water~~ icecream at a rate of 5 cubic feet per minute, find the rate of change of the depth of the ~~water~~ icecream when it is 3 feet deep. The volume of a cone is given by $= \frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height. Give an exact answer in terms of π.



$\frac{dh}{dt} = ?$ when $h = 3f$, $\frac{dV}{dt} = 5\,\frac{ft^3}{min}$

$$V = \frac{1}{3}\pi r^2 h \qquad \frac{r}{h} = \frac{1}{2}$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h \qquad r = \frac{h}{2}$$

$$V = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{4}h^2} = \frac{5}{\frac{\pi}{4}(3^2)} = \boxed{\frac{20}{9\pi}\,\frac{ft}{min}}$$