HW #10 due Tues. 9/29:
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

HW #11 due Fri. 10/2:
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

HW #12 due Mon. 10/5
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem

HW #13 due Wed. 10/7
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity

Test #3

Part A Friday, 10/09
2.5-2.6
Implicit Differentiation
& Related Rates

Part B Monday, 10/12
3.1-3.4
Extrema, Concavity,
Mean Value Theorem

$$f(x) = 2\sin x + \sin 2x$$

$$f'(x) = 2\cos x + \cos 2x \cdot 2$$

$$f''(x) = -2\sin x - 4\sin 2x$$

$$-2(\sin x + 2\sin 2x) = 0$$

$$\sin x + 4\sin x\cos x = 0$$

$$\sin x(1 + 4\cos x) = 0$$

$$\sin x = 0 \qquad \cos x = \frac{-1}{4}$$

$$x = \pi \qquad x = \cos^{-1}\left(\frac{-1}{4}\right)$$

Find y' in terms of x and y.

5. $y = \sin(xy)$

$$y' = [\cos(xy)](y + xy')$$

$$y' = y\cos(xy) + xy'\cos(xy)$$

$$y' - xy'\cos(xy) = y\cos(xy)$$

$$y'(1 - x\cos(xy)) = y\cos(xy)$$

$$y' = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

The radius of a sphere is expanding at a rate of 3 centimeters per second. Find the rate of change of the volume of the sphere when the radius is 12 centimeters.

$$\frac{dV}{dt} = ? \quad \frac{dr}{dt} = 3\,cm/s, \quad r = 12\,cm$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 4\pi(12cm)^2 \cdot (3\,cm/s)$$

$$= \boxed{1728\pi\ cm^3/s}$$

2. $x^3 + y^2 = 10$

a. Find y' in terms of x and y.

b. Find y'' in terms of x and y.

$3x^2 + 2yy' = 0$

$2yy' = -3x^2$

$y' = \dfrac{-3x^2}{2y}$

$y'' = \dfrac{2y(-6x) - (-3x^2)(2y')}{(2y)^2}$

$y'' = \dfrac{-12xy + 6x^2\left(\frac{-3x^2}{2y}\right)}{4y^2}$

1. Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

$f'(x) = 3x^2 - 3x$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

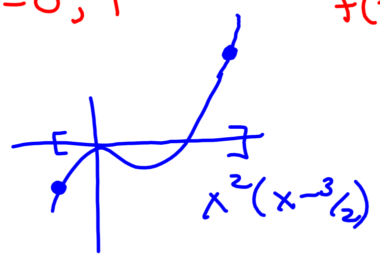
$x = 0, 1$

$f(-1) = -1 - \frac{3}{2} = \frac{-5}{2} \leftarrow$ abs min

$f(0) = 0$

$f(1) = 1 - \frac{3}{2} = \frac{-1}{2}$

$f(2) = 8 - 6 = 2 \leftarrow$ abs max



2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

$$f(x) = (x-3)(x+1)^2 , \qquad [-1, 3]$$

f cts on $[-1,3]$? yes
f diff. on $(-1,3)$? yes
$f(a)=f(b)$? yes
$f(-1)=0$
$f(3)=0$

Rolle's Thm applies

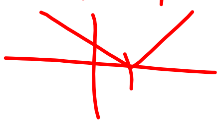
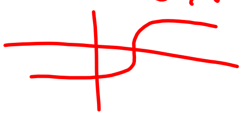
$$f(x) = (x-3)(x+1)^2$$
$$= (x-3)(x^2+2x+1)$$
$$= x^3+2x^2+x-3x^2-6x-3$$
$$f(x) = x^3-x^2-5x-3$$

$$f'(x) = 3x^2-2x-5$$
$$3x^2-2x-5=0$$
$$(3x-5)(x+1)=0$$
$$\boxed{x=5/3}, \cancel{x=-1}$$

$$3x^2+3x-5x-5=0$$
$$3x(x+1)-5(x+1)=0$$
$$(x+1)(3x-5)=0$$

examples of continuous but not differentiable functions

$f(x) = |x-2|$ $\qquad \sqrt[3]{x-2}$

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2 - x - 2)$, $[-1, 1]$

f cts on $[-1,1]$? yes
f diff on $(-1,1)$? yes
MVT applies

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{1-1-2-(-1-1+2)}{2} = \frac{-2}{2} = -1$$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\cancel{x=1}$, $\boxed{x = -1/3}$

5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

$$y' = \frac{(x^2-9)(2x) - x^2(2x)}{(x^2-9)^2}$$

$$= \frac{2x^3 - 18x - 2x^3}{(x^2-9)^2} = \frac{-18x}{(x^2-9)^2}$$

critical #'s : 0, 3, -3

$f'(-4)$ $f'(-1)$ $f'(1)$ $f'(4)$

+ -3 + 0 − 3 −

↗ ↗ ↘ ↘

no absolute extrema

f is increasing on $(-\infty, -3) \cup (-3, 0)$
f is decreasing on $(0, 3) \cup (3, \infty)$
f has a relative max @ $(0, 0)$

7. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = \frac{x}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)(2(x^2+1)\cdot 2x)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)(-2x) - 4x(1-x^2)}{(x^2+1)^3}$$

$x^2 = 3$

$x = \pm\sqrt{3}$

$$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = \frac{2x^3 - 6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

$x = 0, \sqrt{3}, -\sqrt{3}$

$-2 \quad -1 \quad 1 \quad 2$

$- \;\; -\sqrt{3} \;\; + \;\; 0 \;\; - \;\; \sqrt{3} \;\; +$

$\cap \quad \cup \quad \cap \quad \cup$

f is concave up on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

f is concave down on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

f has inflection points @ $\left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right)$, $(0, 0)$ & $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$