Homework #15 - due Fri, 10/16

- 3.5 #15-31odd - limits at infinity
- 3.7 #3,5,17,23,29 - optimization

Homework #16 - due Tues, 10/20
- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs

Test #4 - Tues/Wed/Fri 10/20,21,23?

3.5 Limits at Infinity

$\lim_{x \to \infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x) = \frac{5x^2 - 3x + 4}{2x^2 + 5x} \approx \frac{5x^2}{2x^2} = \frac{5}{2}$$

Horizontal asymptote: $y = 5/2$

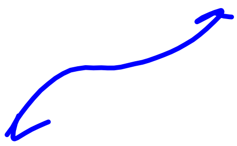
$$\lim_{x \to -\infty} f(x) = 5/2 \quad \& \quad \lim_{x \to \infty} f(x) = 5/2$$

$$f(x) = \frac{2x - 4}{3x^4} \approx \frac{2x}{3x^4} = \frac{2}{3x^3}$$

Horizontal asymptote: $y = 0$

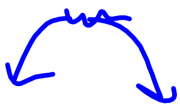
$$\lim_{x \to -\infty} f(x) = 0 \quad \& \quad \lim_{x \to \infty} f(x) = 0$$

$$f(x) = \frac{2x^7 - 4x^3 - 2}{5x^4 + 1} \approx \frac{2x^7}{5x^4} = \frac{2}{5}x^3$$

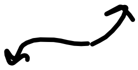
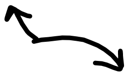
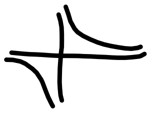
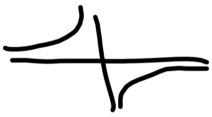
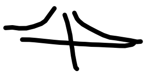
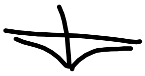


$$\lim_{x \to -\infty} f(x) = -\infty \quad \& \quad \lim_{x \to \infty} f(x) = \infty$$

$$f(x) = \frac{2 - 7x^3 + 2x}{1 + x} \approx \frac{-7x^3}{x} = -7x^2$$



$$\lim_{x \to -\infty} f(x) = -\infty \quad \& \quad \lim_{x \to \infty} f(x) = -\infty$$

f(x) (ratio of lead term)	picture	$\lim_{x\to-\infty} f(x)$	$\lim_{x\to\infty} f(x)$
$+x^{even}$		∞	∞
$-x^{even}$		$-\infty$	$-\infty$
$+x^{odd}$		$-\infty$	∞
$-x^{odd}$		∞	$-\infty$
c		c	c
$+\frac{1}{x^{odd}}$		0	0
$-\frac{1}{x^{odd}}$		0	0
$+\frac{1}{x^{even}}$		0	0
$-\frac{1}{x^{even}}$		0	0

24. $\lim_{x\to-\infty}\left(\frac{1}{2}x - \frac{4}{x^2}\right)$

$= \lim_{x\to-\infty} \frac{1}{2}x - \lim_{x\to-\infty} \frac{4}{x^2}$

$= -\infty - 0$

$= \boxed{-\infty}$

$= \lim_{x\to-\infty} \frac{x^3-8}{2x^2}$

$= \lim_{x\to-\infty} \frac{x^3}{2x^2}$

$= \lim_{x\to-\infty} \frac{x}{2}$

$= \boxed{-\infty}$

26. $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$

$= \lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2}}$

$= \lim\limits_{x\to-\infty} \dfrac{x}{|x|}$

$= \lim\limits_{x\to-\infty} \dfrac{x}{-x} = \lim\limits_{x\to-\infty} -1$

$= \boxed{-1}$

$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

* Two horizontal asymptotes $y=-1$ & $y=1$

$\lim\limits_{x\to\infty} \dfrac{5x-2}{\sqrt{9x^2+3}} = \lim\limits_{x\to\infty} \dfrac{5x}{\sqrt{9x^2}}$

$= \lim\limits_{x\to\infty} \dfrac{5x}{3|x|}$

$= \lim\limits_{x\to\infty} \dfrac{5x}{3x} = \lim\limits_{x\to\infty} \dfrac{5}{3}$

$= 5/3$

2 H.A.'s $y=5/3$ & $y=-5/3$

30. $\lim_{x\to\infty} \frac{x - \cos x}{x}$

$= \lim_{x\to\infty} \frac{x}{x} - \lim_{x\to\infty} \frac{\cos x}{x}$ — bounded $[-1, 1]$; $x \to \infty$

$= \lim_{x\to\infty} 1 - 0$

$\frac{\text{bounded}}{\infty} \to 0$

$= \boxed{1}$

32. $\lim_{x\to\infty} \cos\frac{1}{x} = \lim_{x\to\infty} \cos\left[\frac{1}{x}\right]$

$= \cos\left[\lim_{x\to\infty} \frac{1}{x}\right]$

$= \cos 0$

$= \boxed{1}$

18. c.

$$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}}$$

$$= \lim_{x\to\infty} \frac{5}{4}x^{1}$$

$$= \boxed{\infty}$$

3.7 Optimization Problems

4. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

x & y

$xy = 192 \qquad x = \frac{192}{y}$

$S(x,y) = x + 3y$

$S(y) = \frac{192}{y} + 3y = 192y^{-1} + 3y$

$S'(y) = -192y^{-2} + 3$

$0 = \frac{-192}{y^2} + 3$

$\frac{192}{y^2} = 3$

$192 = 3y^2$

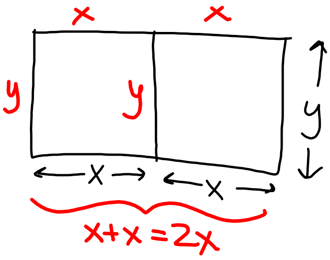
$64 = y^2$

$\boxed{8 = y}$

$x = \frac{192}{y} = \frac{192}{8}$

$\boxed{x = 24}$

18. A rancher has 200 feet of fencing (perimeter) with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



$200 = 4x + 3y \Rightarrow 3y = 200 - 4x$

$y = \frac{200}{3} - \frac{4x}{3}$

$A(x,y) = 2xy$

$A(x) = 2x\left(\frac{200}{3} - \frac{4x}{3}\right)$

$A(x) = \frac{400}{3}x - \frac{8}{3}x^2$

$A'(x) = \frac{400}{3} - \frac{16}{3}x$

$\frac{400}{3} - \frac{16}{3}x = 0$

$\frac{400}{3} = \frac{16}{3}x$

$\frac{3}{16} \cdot \frac{400}{3} = x$

$25 = x$ ft

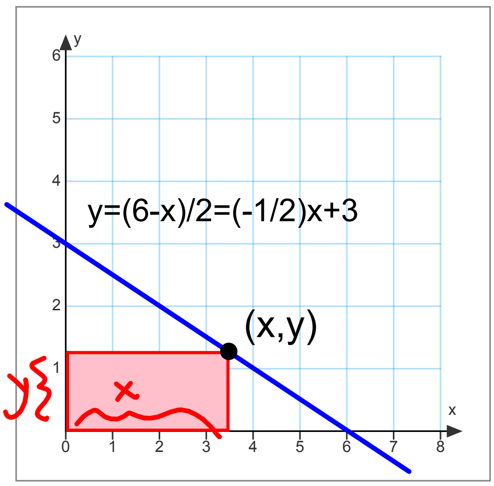
$y = \frac{200}{3} - \frac{4x}{3}$

$= \frac{200}{3} - \frac{4(25)}{3}$

$= \frac{200}{3} - \frac{100}{3}$

$y = \frac{100}{3}$ ft

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$y = \frac{6-x}{2} = \frac{6}{2} - \frac{x}{2} = 3 - \frac{x}{2}$

$y = -\frac{1}{2}x + 3$

$A(x,y) = xy$

$A(x) = x\left(-\frac{1}{2}x + 3\right)$

$A(x) = -\frac{1}{2}x^2 + 3x$

$A'(x) = -x + 3$

$-x + 3 = 0$

$x = 3$

$y = -\frac{1}{2}(3) + 3$

$y = \frac{3}{2}$