Homework #15 - due Fri, 10/16

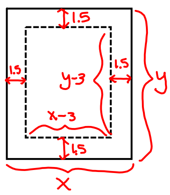
- 3.5 #15-31odd - limits at infinity
- 3.7 #3,5,17,23,29 - optimization

Homework #16 - due Tues, 10/20

- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs

Test #4 - Tues/Wed/Fri 10/20,21,23?

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



$A(x,y) = xy$

$(x-3)(y-3) = 36$

$x - 3 = \frac{36}{y-3}$

$x = \frac{36}{y-3} + 3$

$A(y) = y\left(\frac{36}{y-3} + 3\right)$

$A(y) = \frac{36y}{y-3} + 3y$

$A'(y) = \frac{(y-3)(36) - 1(36y)}{(y-3)^2} + 3$

$= \frac{36y - 3(36) - 36y}{(y-3)^2} + 3$

$0 = \frac{-3(36)}{(y-3)^2} + 3$

$\frac{3(36)}{(y-3)^2} = 3$

$\frac{36}{(y-3)^2} = 1$

$36 = (y-3)^2$

$\pm 6 = y - 3$

$3 \pm 6 = y$

$y = 9, \cancel{-3}$

$\boxed{y = 9 \text{ in}}$

$x = \frac{36}{9-3} + 3$

$\boxed{x = 9 \text{ in}}$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\boxed{\frac{0}{0}, \frac{\infty}{\infty},}$ $0 \cdot \infty$, 1^{∞}, 0^0, *and* $\infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

7.7

12. $\lim_{x \to -1} \frac{x^2 - x - 2}{x+1} = \lim_{x \to -1} \frac{\cancel{(x+1)}(x-2)}{\cancel{x+1}} = \boxed{-3}$

plugging in -1 yields $\frac{0}{0}$, so l'hopital's rule applies

$= \lim_{x \to -1} \frac{2x-1}{1} = 2(-1) - 1 = \boxed{-3}$

16. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} = \frac{e^0-1-0}{0^3} = \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{e^x-1}{3x^2}$

$= \lim_{x\to 0^+} \frac{e^x}{6x} = \boxed{\infty}$

18. $\lim_{x\to 1} \frac{\ln(x^2)}{x^2-1} = \frac{0}{0}$

$\log_a(1)=0$

$a^0=1$

$= \lim_{x\to 1} \frac{\frac{1}{x^2}\cdot\cancel{2x}}{\cancel{2x}}$

$= \lim_{x\to 1} \frac{1}{x^2} = \boxed{1}$

20. $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx} = \lim\limits_{x\to 0} \underbrace{\dfrac{\sin ax}{ax}}_{1} \cdot \underbrace{\dfrac{bx}{\sin bx}}_{1} \cdot \dfrac{a}{b} = \dfrac{a}{b}$

$= \dfrac{0}{0}$

$= \lim\limits_{x\to 0} \dfrac{a\cos ax}{b\cos bx} = \dfrac{a\cdot 1}{b\cdot 1} = \boxed{\dfrac{a}{b}}$

28. $\lim\limits_{x\to\infty} \dfrac{x^2}{e^x} = \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{2x}{e^x} = \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{2}{e^x} = \boxed{0}$

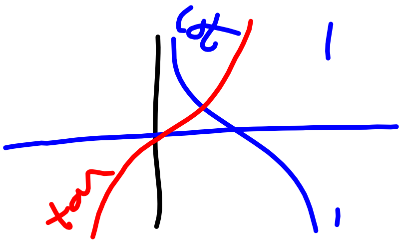
36. $\lim_{x\to\infty} \frac{e^{x/2}}{x} = \frac{\infty}{\infty}$

$\frac{x}{2} = \frac{1}{2}x$

$= \lim_{x\to\infty} \frac{e^{x/2}\cdot\frac{1}{2}}{1} = \boxed{\infty}$

38. $\lim_{x\to 0^+} x^3 \cot x$

$= 0 \cdot \infty$



$0 \approx \frac{1}{\infty}$

$\infty \approx \frac{1}{0}$

$\cot x = \frac{1}{\tan x}$

$= \lim_{x\to 0^+} \frac{x^3}{\tan x} = \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{3x^2}{\sec^2 x} = \frac{0}{1} = \boxed{0}$

40. $\lim_{x\to\infty} x\tan\frac{1}{x} = \infty \cdot 0$

$= \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0}$

$= \lim_{x\to\infty} \frac{\sec^2\frac{1}{x}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \boxed{1}$

42. $\lim_{x\to 0^+} (e^x + x)^{2/x} = 1^\infty$

$y = \lim_{x\to 0^+} (e^x + x)^{2/x}$

$\ln y = \ln\left[\lim_{x\to 0^+} (e^x + x)^{2/x}\right]$