HW #16 - due Wed, 10/21
- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs

Test #4 - Fri 10/23

on sections 3.5, 3.7, and 7.7 with some review

Final Exam:

2nd per - Tues 10/27 9-11am
4th per - Wed 10/28 1-3pm

Lowest of 4 regular test grades will be dropped (if it helps you)

Lowest quiz & HW will be dropped

Final Exam can replace 2nd lowest test grade (if it helps you)

$$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^n}$$

$$= \lim_{x\to 0^+} \frac{e^x-1}{nx^{n-1}} = 0 \text{ if } n=1,$$

otherwise $\frac{0}{0}$ & L'H...

$$\lim_{x\to 0^+} \frac{e^x}{n(n-1)x^{n-2}} = \frac{1}{2} \text{ if } n=2$$

otherwise

$$= \infty \text{ if } n \geq 3$$

$$\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x} = \lim_{x\to 0} \frac{(4-x^2)^{1/2}-2}{x}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cancel{2}}(4-x^2)^{-1/2}\cdot(-\cancel{2}x)}{1}$$

$$= \lim_{x\to 0} \frac{-x}{\sqrt{4-x^2}} = 0$$

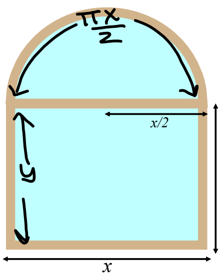
$$\lim_{x\to\infty} \frac{x^3}{e^{x/2}}$$

$$\frac{3x^2}{e^{x/2}\cdot\frac{1}{2}}$$

$$\frac{6x}{e^{x/2}\frac{1}{2}\cdot\frac{1}{2}}$$

$$\lim_{x\to\infty} \frac{6}{e^{x/2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}} = 0$$

3.7 #23 - A **Norman window** is constructed by adjoining a semi-circle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.



$$A(x,y) = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$16 = 2y + x + \frac{\pi}{2}x$$

$$16 - x - \frac{\pi}{2}x = 2y$$

$$8 - \frac{1}{2}x - \frac{\pi}{4}x = y$$

$$A(x) = x\left(8 - \frac{1}{2}x - \frac{\pi}{4}x\right) + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$A(x) = 8x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$$

$$A(x) = 8x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$$

$$A'(x) = 8 - x - \frac{\pi}{4}x$$

$$8 - x - \frac{\pi}{4}x = 0$$

$$8 = x + \frac{\pi}{4}x$$

$$8 = x\left(1 + \frac{\pi}{4}\right)$$

$$\frac{8}{1+\frac{\pi}{4}} = x$$

$$\frac{8}{\frac{4+\pi}{4}} = x$$

$$\boxed{x = \frac{32}{4+\pi}}$$

$$y = 8 - \frac{1}{2}x - \frac{\pi}{4}x$$

$$y = 8 - \frac{1}{\cancel{2}}\cdot\frac{\overset{16}{\cancel{32}}}{4+\pi} - \frac{\pi}{\cancel{4}}\cdot\frac{\overset{8}{\cancel{32}}}{4+\pi}$$

$$y = \frac{8(4+\pi)}{4+\pi} - \frac{16}{4+\pi} - \frac{8\pi}{4+\pi}$$

$$= \frac{32 + 8\pi - 16 - 8\pi}{4+\pi}$$

$$y = \frac{16}{4+\pi}$$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\boxed{\frac{0}{0}, \frac{\infty}{\infty}}$, $0 \cdot \infty$, 1^∞, 0^0, *and* $\infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0, \infty/\infty, (-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

$$\lim_{x \to \infty} \sin \frac{1}{x}$$

$$= \sin\left[\lim_{x\to\infty} \frac{1}{x}\right]$$

$$= \sin 0$$

$$= \boxed{0}$$

44. $$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \qquad = 1^{\infty}$$

$$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$$\ln y = \ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right]$$

$$\ln y = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x$$

$$\ln y = \lim_{x\to\infty} \frac{x}{1}\ln\left(1+\frac{1}{x}\right) \qquad = \infty \cdot 0 = \frac{\infty}{\left(\frac{1}{0}\right)} = \frac{0}{\left(\frac{1}{\infty}\right)}$$

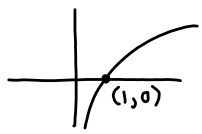
$$\ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \qquad = \frac{0}{0}$$ L'Hopital applies

$$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\frac{-1}{x^2}}}$$

$$\ln y = \frac{1}{1+0} = 1$$

$$e^{\ln y} = e^1$$

$$y = \boxed{e}$$



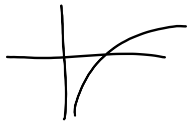
$\ln 1 = 0$

$$\lim_{x\to 0^+} (\sin x)^x = 0^0$$

$$y = \lim_{x\to 0^+} (\sin x)^x$$ ← take log, interchange limit & log, apply log rule

$$\ln y = \ln\left[\lim_{x\to 0^+} (\sin x)^x\right]$$

$$\ln y = \lim_{x\to 0^+} \ln\left[(\sin x)^x\right]$$



$$\ln y = \lim_{x\to 0^+} x \ln(\sin x) = 0\cdot(-\infty)$$

$$\ln y = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} = \frac{-\infty}{\infty}$$ L'Hopital applies

$$\ln y = \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{\frac{-1}{x^2}} = \lim \frac{\cot x}{\frac{-1}{x^2}} = \lim \frac{-x^2}{\tan x}$$

$$\ln y = \lim_{x\to 0^+} \frac{-x^2}{\tan x} = \frac{0}{0}$$ L'H applies

$$\ln y = \lim_{x\to 0^+} \frac{-2x}{\sec^2 x}$$

$$\ln y = \frac{0}{1}$$

$$\ln y = 0$$

$$e^{\ln y} = e^0$$

$$\boxed{y = 1}$$

$$\lim_{x\to 0^+} x^{1/x} = 0^{\infty} = \boxed{0}$$

$$1^{1/1} = 1$$

$$\left(\frac{1}{2}\right)^{1/\frac{1}{2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(\frac{1}{10}\right)^{1/\frac{1}{10}} = \frac{1}{10^{10}}$$

$$\left(\frac{1}{1000}\right)^{1/\frac{1}{1000}} = \frac{1}{1000^{1000}}$$

$$\lim_{x\to0}\frac{e^x-(1-x)}{x} = \frac{0}{0}$$

$$\lim_{x\to0}\frac{e^x+1}{1} = \frac{e^0+1}{1} = \frac{1+1}{1} = \boxed{2}$$

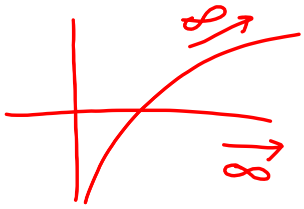
$$\boxed{\begin{array}{l}\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even}\end{cases} \\ |x| = \begin{cases} x, & x\ge 0 \\ -x, & x<0\end{cases}\end{array}}$$

$$\lim_{x\to-\infty}\frac{x^2}{\sqrt{x^2+1}} = \lim_{x\to-\infty}\frac{x^2}{\sqrt{x^2}} = \lim_{x\to-\infty}\frac{x^2}{|x|}$$

$$= \lim_{x\to-\infty}\frac{x^2}{-x} = \lim_{x\to-\infty} -x = -(-\infty) = \boxed{\infty}$$

$$\lim_{x\to\infty}\frac{\sin x \in [-1,1]}{x-\pi \to \infty} = \boxed{0}$$

$$\lim_{x\to\infty}\frac{\ln x^4}{x^3} = \frac{\infty}{\infty} \quad \text{L'H}$$



$$\lim_{x\to\infty}\frac{\frac{1}{x^4}\cdot 4x^3}{3x^2} = \lim_{x\to\infty}\frac{\left(\frac{4x^3}{x^4}\right)}{\left(\frac{3x^2}{1}\right)}$$

$$= \lim_{x\to\infty}\frac{4}{x}\cdot\frac{1}{3x^2}$$

$$= \lim_{x\to\infty}\frac{4}{3x^3} = \boxed{0}$$