HW #16 - due Wed, 10/21

- 7.7 #11-35odd - l'Hopital's rule
- 7.7 #37-53odd - l'Hopital's rule with logs

Test #4 - Fri 10/23

on sections 3.5, 3.7, and 7.7 with some review

Final Exam:

2nd per - Tues 10/27 9-11am
4th per - Wed 10/28 1-3pm

Lowest of 4 regular test grades will be dropped (if it helps you)

Lowest quiz & HW will be dropped

Final Exam can replace 2nd lowest test grade (if it helps you)

$$\lim_{x\to 1^+} (\ln x)^{x-1} = 0^0$$

$$\ln y = \lim_{x\to 1^+} (x-1)\ln(\ln x) = 0\cdot(-\infty)$$

$$= \lim \frac{\ln(\ln x)}{\frac{1}{x-1}}$$

$$\ln y = \lim \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{\frac{-1}{(x-1)^2}} = \lim \frac{-(x-1)^2}{x\ln x}$$

$$\ln y = \lim_{x\to 1^+} \frac{-2(x-1)}{1\cdot\ln x + x\cdot\frac{1}{x}} = \lim_{x\to 1^+} \frac{-2x+2}{\ln x + 1} = \frac{0}{1}$$

$$y = e^0 = \boxed{1}$$

$$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^n} \qquad n \in \mathbb{Z}^+$$

$$= \lim_{x\to 0^+} \frac{e^x-1}{nx^{n-1}} \qquad = 0 \text{ if } n=1$$

$$= \frac{0}{0} \text{ if } n \geq 2$$

so L'H

$$= \lim_{x\to 0^+} \frac{e^x}{n(n-1)x^{n-2}} \qquad = \frac{1}{2\cdot1\cdot1} = \frac{1}{2} \text{ if } n=2$$

$$= \infty \text{ if } n \geq 3$$

$$\lim_{x\to\infty} \frac{2x^3-5x+1}{4x^3-3x^2+x+25} = \frac{1}{2}$$

$$= \lim_{x\to\infty} \frac{2x^3}{4x^3} = \lim_{x\to\infty} \frac{1}{2}$$

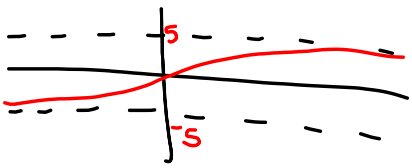
$$\lim_{x\to-\infty} \frac{-2x+5}{\sqrt{x^2+2x}} = \lim_{x\to-\infty} \frac{-2x}{\sqrt{x^2}} = \lim_{x\to-\infty} \frac{-2x}{|x|}$$

$$= \lim_{x\to-\infty} \frac{-2x}{-x} = \boxed{2}$$

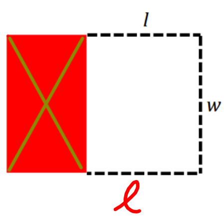
Find the horizontal asymptotes. $f(x) = \frac{5x}{\sqrt{x^2+5}}$ $\approx \frac{5x}{\sqrt{x^2}} = \frac{5x}{|x|}$

$y = 5$ & $y = -5$

$= \begin{cases} 5, & x>0 \\ -5, & x<0 \end{cases}$



If I have 200 meters of fence to make a rectangular yard attached to the side of a barn, what dimensions will yield the maximum area?



$A(\ell, w) = \ell w$

$200 = 2\ell + w \Rightarrow \underline{w = 200 - 2\ell}$

$A(\ell) = \ell(200 - 2\ell)$

$A(\ell) = 200\ell - 2\ell^2$

$A'(\ell) = 200 - 4\ell$

$w = 200 - 2(50)$

$\boxed{w = 100\ m}$

$200 - 4\ell = 0$

$200 = 4\ell$

$\boxed{50_m = \ell}$

1. Use the product rule to differentiate the function.

a. $f(x) = (x^2 + \sin x)(3x - 15)$

b. $f(x) = \sqrt{x}\tan x$ $= x^{1/2}\tan x$

a. $(2x+\cos x)(3x-15) + (x^2+\sin x)(3)$

b. $\frac{1}{2}x^{-1/2}\tan x + x^{1/2}\sec^2 x$

c. $f(x) = 3x - 5\cot(\pi x)^2$

d. $f(x) = \ln(\tan^{-1}(2x))$

e. $f(x) = 5^{\csc x}\sqrt{x^3 - 7x}$ $= (5^{\csc x})\cdot(x^3-7x)^{1/2}$

$f'(x) = (x^3-7x)^{1/2}\cdot 5^{\csc x}\ln 5\cdot(-\csc x\cot x) + 5^{\csc x}\cdot\frac{1}{2}(x^3-7x)^{-1/2}\cdot(3x^2-7)$

6. Find an equation of the tangent line to the graph of f at the indicated point.

$f(x) = \sqrt{2x^2 - 7}$, $(2, 1)$

$= (2x^2-7)^{1/2}$

$y-1 = 4(x-2)$

$y = 4x - 8 + 1$

$\boxed{y = 4x - 7}$

$f'(x) = \frac{1}{2}(2x^2-7)^{-1/2} \cdot 4x = \frac{2x}{\sqrt{2x^2-7}}$

$m = f'(2) = \frac{2(2)}{\sqrt{2(2)^2-7}} = \frac{4}{1} = 4$

7. The length of a rectangle is given by $2(t^2+1)$ and its height is $\sqrt{t+5}$ where t is time in seconds and dimensions are in centimeters.

$A(t) = 2(t^2+1)\sqrt{t+5}$

a. Find the average rate of change of the area from time 4 to time 11.

$\frac{A(11)-A(4)}{11-4} = \frac{2(11^2+1)\sqrt{11+5} - 2(4^2+1)\sqrt{4+5}}{7}$

$= \frac{2(122)(4) - 2(17)(3)}{7} = \boxed{}$

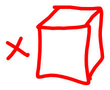
b. Find the instantaneous rate of change of the area at time 4.

$A'(t) =$

$A'(4) = \boxed{}$

10. All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 10 centimeters?

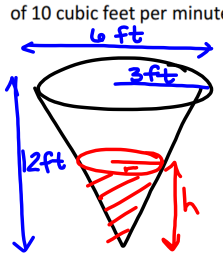
$V_{cube} = x^3$ $\quad \frac{dx}{dt} = 3 \text{ cm/s}$ $\quad \frac{dV}{dt} = ?$ when $x = 10$



$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$= 3(10)^2 \cdot 3 = \boxed{900 \ \tfrac{\text{cm}^3}{s}}$$

11. A conical tank (with vertex down) is 6 feet across and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.



$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$; $\frac{dh}{dt} = ?$ when $h = 8$ ft

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 \cdot h = \frac{\pi}{12} h^3$

$\frac{r}{h} = \frac{3}{12}$

$r = \frac{h}{4}$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \cdot \frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{4} h^2} = \frac{10}{\frac{\pi}{4} \cdot 8^2} = \frac{10}{16\pi} = \boxed{\frac{5}{8\pi} \ \text{ft/min}}$$

13. Find the limit.

a. $\lim_{x\to0}\frac{\sin x\,(1-\cos x)}{2x^2}$ $=\lim_{x\to0}\frac{\sin x}{x}\cdot\frac{1-\cos x}{x}\cdot\frac{1}{2}$ $=\boxed{0}$

b. $\lim_{x\to3^-} f(x)$, where $f(x)=\begin{cases}\frac{x+2}{2}, x\le3\\ \frac{12-2x}{3}, x>3\end{cases}$ $=5/2$

c. $\lim_{x\to3^+}\frac{x^2}{x^2-9}$ $=\infty$

d. $\lim_{x\to\infty}\frac{3x^3+2}{9x^3-2x^2+7}$ $=\frac{1}{3}$

$\lim_{x\to0}\frac{\cos x(1-\cos x)+\sin x(\sin x)}{4x}$

$=\lim_{x\to0}\frac{\cos x-\cos^2 x+\sin^2 x}{4x}$

$=\lim_{x\to0}\frac{-\sin x-2\cos x(-\sin x)+2\sin x\cos x}{4}$ ~~$=\lim_{x\to0}\frac{\cos x-\cos 2x}{4x}$~~

$=\lim_{x\to0}\frac{-\sin x+4\sin x\cos x}{4}$ $=\boxed{0}$

$\lim_{x\to0}(\cos x)^{1/x}=1^\infty$

$\ln y=\lim_{x\to0}\left[\frac{1}{x}\ln(\cos x)\right]=\frac{0}{0}$

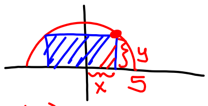
$\ln y=\lim_{x\to0}\frac{\frac{1}{\cos x}\cdot(-\sin x)}{1}=0$

$y=e^0=\boxed{1}$

14. Find the derivative of the function using the definition (limit of the difference quotient)

$f(x) = x^3 - 12x$

15. A rectangle is bounded by the x-axis and the semi-circle $y = \sqrt{25 - x^2}$ (see figure on p. 217 of textbook). What length and width should the rectangle have so that its area is a maximum?



$$x^2 + y^2 = 25$$
$$y^2 = 25 - x^2$$
$$y = \pm\sqrt{25 - x^2}$$

$$A(x, y) = 2x \cdot y$$
$$A(x) = 2x\sqrt{25 - x^2}$$
$$A(x) = (2x)(25 - x^2)^{1/2}$$
$$A'(x) = 2(25 - x^2)^{1/2} + \cancel{2}x\left(\tfrac{1}{\cancel{2}}(25 - x^2)^{-1/2}(-2x)\right)$$
$$2\sqrt{25 - x^2} - \frac{2x^2}{\sqrt{25 - x^2}} = 0$$
$$2(25 - x^2) - 2x^2 = 0$$
$$50 - 2x^2 - 2x^2 = 0$$
$$50 = 4x^2$$
$$\frac{25}{2} = x^2$$
$$\frac{5}{\sqrt{2}} \pm = x$$

width of rectangle

$$\frac{2 \cdot 5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{5\sqrt{2}}$$

heigh of rectangle

$$y = \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{\frac{25 \cdot 2}{2} - \frac{25}{2}}$$
$$= \sqrt{\frac{25}{2}}$$
$$= \boxed{\frac{5}{\sqrt{2}}}$$