Diff Cal - Test 4 Solutions & Final Exam Review

October 26, 2015

Final Exam:

2nd per - Tues 10/27 9-11am
4th per - Wed 10/28 1-3pm

Lowest of 4 regular test grades will be dropped (if it helps you)

Lowest quiz & HW will be dropped

Final Exam can replace 2nd lowest test grade (if it helps you)

1. $\lim_{x\to -4} \frac{x^2+x-12}{x+4} = \lim \frac{\cancel{(x+4)}(x-3)}{\cancel{x+4}} = -4-3 = \boxed{-7}$

2. $\lim_{x\to \infty} \frac{2x+6}{x^2-2x-15} = \lim \frac{2x}{x^2} = \lim_{x\to\infty} \frac{2}{x} = \boxed{0}$

3. $\lim_{x\to -\infty} \frac{8x^3-2x+1}{4x^3-2x^2+6x-3} = \lim \frac{8\cancel{x^3}}{4\cancel{x^3}} = \boxed{2}$

4. $\lim_{x\to\infty} \frac{x-\cos x}{x}$

$= \lim \frac{x}{x} - \lim \frac{\cos x}{x}$

$= 1 - \frac{[-1,1]}{\infty}$

$= 1 - 0 = \boxed{1}$

5. $\lim_{x\to-\infty} \frac{-2x}{\sqrt{x^2+9}} = \lim \frac{-2x}{\sqrt{x^2}} = \lim \frac{-2x}{|x|}$

$= \lim \frac{-2x}{-x} = \boxed{2}$

6. $\lim\limits_{x\to 0}\frac{e^x-x-1}{2x^2} = \lim\limits_{x\to 0}\frac{e^x-1}{4x} = \lim\limits_{x\to 0}\frac{e^x}{4} = \frac{e^0}{4} = \boxed{\frac{1}{4}}$

7. $\lim\limits_{x\to 1}\frac{\ln(x^2)}{x^2-1} = \lim\limits_{x\to 1}\frac{\frac{1}{x^2}\cdot \cancel{2x}}{\cancel{2x}} = \frac{1}{1^2} = \boxed{1}$

8. $\lim_{x\to 0}(e^x + x)^{2/x} = y$

$$\ln y = \lim_{x\to 0} \frac{2}{x} \ln(e^x + x)$$

$$\ln y = \lim_{x\to 0} \frac{2\ln(e^x + x)}{x}$$

$$\ln y = \lim_{x\to 0} \frac{2 \cdot \frac{1}{e^x + x} \cdot e^x + 1}{1}$$

$$\ln y = \lim_{x\to 0} \frac{2(e^x + 1)}{e^x + x}$$

$$\ln y = \frac{2(e^0 + 1)}{e^0 + 0} = \frac{2(1+1)}{1} = \frac{4}{1}$$

$$\ln y = 4$$

$$e^{\ln y} = e^4$$

$$\boxed{y = e^4}$$

9. Find two positive numbers that satisfy the given requirements: the product is 324 and the sum of the first plus four times the second is a minimum.

$$xy = 324$$

$$S(x,y) = x + 4y$$

$$S(y) = \frac{324}{y} + 4y$$

$$S'(y) = \frac{-324}{y^2} + 4$$

$$x = \frac{324}{y}$$

$$\boxed{x = 36}$$

$$\frac{-324}{y^2} + 4 = 0$$

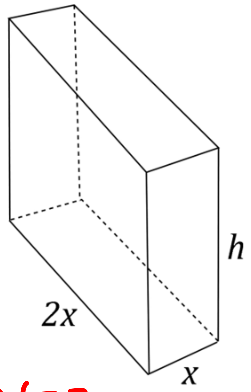
$$\frac{-324}{y^2} = -4$$

$$81 = \frac{324}{4} \quad y^2$$

$$\boxed{y = 9}$$

10. maximize the volume

surface area is 432 cm^2 $\quad 2(xh)+2(2x^2)+2(2xh)$.



$432 = 2xh + 4x^2 + 4xh$

$432 = 6xh + 4x^2$

$432 - 4x^2 = 6xh$

$\frac{432-4x^2}{6x} = h$

$\boxed{h=8}$

$V = 2x \cdot x \cdot h$

$V = 2x^2h$

$V(x) = \frac{\cancel{2}x^{\cancel{2}}}{1} \cdot \frac{432-4x^2}{{}_3\cancel{6x}}$

$V(x) = \frac{432}{3}x - \frac{4}{3}x^3$

$V'(x) = 144 - 4x^2$

$144 - 4x^2 = 0$

$x^2 = \frac{144}{4} = 36$

$x = 6$

can height and diameter

minimize can surface area

$A = 2\pi r^2 + 2\pi rh$,

355 cm^3 volume

$V = \pi r^2 h = 355$

$h = \frac{355}{\pi r^2}$

$A(r) = 2\pi r^2 + 2\cancel{\pi}\cancel{r} \cdot \frac{355}{\cancel{\pi} r^{\cancel{2}}}$

$A(r) = 2\pi r^2 + \frac{710}{r}$

$A'(r) = 4\pi r - \frac{710}{r^2}$

$4\pi r = \frac{710}{r^2}$

$r^3 = \frac{710}{4\pi}$

$r = \sqrt[3]{\frac{710}{4\pi}}$

$$\boxed{\begin{aligned} d &= 2r \approx 7.67 \\ h &= \frac{355}{\pi r^2} \approx 7.67 \end{aligned}}$$

$355 = \pi r^2 h \qquad 2r = h$

$355 = \pi r^2(2r)$

$\frac{355}{2\pi} = r^3 \qquad r = \sqrt[3]{\frac{355}{2\pi}}$

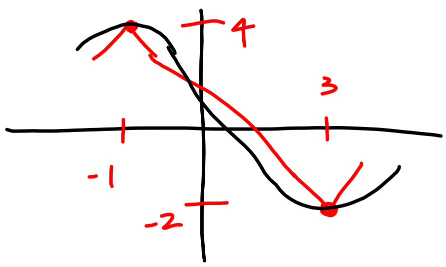
16 Hw $\quad \dfrac{a}{160}$

5 Q $\quad \dfrac{b}{\begin{matrix}0\\5\end{matrix}}$

3 T $\quad$ Top2 + $c/300$

F $\quad 200 = 2 \cdot c$

$$\frac{a + b + \text{top2} + c + 2c}{710} = 0.9$$



$$x = \pm\sqrt{-4}$$
$$= \pm 2i$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h} = f'\left(\frac{1}{2}\right)$$

$$f(x) = 8x^8$$

$$f'(x) = 64x^7$$

$$f'\left(\tfrac{1}{2}\right) = 64\left(\tfrac{1}{2}\right)^7 = \frac{2^6}{2^7} = \boxed{\frac{1}{2}}$$

$$\frac{dA}{dt} = 2\frac{dr}{dt} \qquad A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\cancel{2}\cancel{\frac{dr}{dt}} = \cancel{2}\pi r \cancel{\frac{dr}{dt}}$$

$$1/\pi = r$$

17 $y = 5x^4 - x^5$

$y' = 20x^3 - 5x^4$

$y'' = 60x^2 - 20x^3$

$0 = 20x^2(3 - x)$ $x = 0,\ x = 3$

$\boxed{(0,0)\ \&\ (3,162)}$

$\sin x = e^y \longrightarrow \ln(\sin x) = y \ln e$

$\cos x = e^y \cdot y'$ $\frac{1}{\sin x} \cdot \cos x = y'$

$\frac{\cos x}{e^y} = y'$ $\boxed{\cot x} =$

$\frac{\cos x}{\sin x} = \boxed{\cot x}$

ε-δ def of limit.

$\lim_{x \to c} f(x)$ is L if

given $\varepsilon > 0$, there exists a $\delta > 0$ such that if $|x-c| < \delta$, then $|f(x) - L| < \varepsilon$

(or $|f(x)-L| < \varepsilon$ wherever $|x-c| < \delta$)

$|f(x) - L| = |1 - 3x - (-5)| = |6 - 3x| = 3|2-x|$
$= 3|x-2|$

$3|x-2| < \varepsilon$

$|x-2| < \varepsilon/3 = \delta$

$y' = (x+1) \cdot \frac{1}{1+x^2} + 1 \cdot \arctan x$

$y' = \frac{x+1}{1+x^2} + \arctan x$

$y'' = \frac{(1+x^2) \cdot 1 - (x+1) \cdot 2x}{(1+x^2)^2} + \frac{1}{1+x^2}$

$0 = \frac{1 + \cancel{x^2} - \cancel{2x^2} - 2x + 1 + \cancel{x^2}}{(1+x^2)^2}$

$0 = \frac{2-2x}{(1+x^2)^2} = \frac{2(1-x)}{(1+x^2)^2}$

$x = 1$

$y = (1+1) \arctan 1$
$= 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$

$$\left[f(x^3)\right]'' = ?$$

$$f'(x) = g(x)$$

$$g'(x) = f(x^2)$$

$$\left(f(x^3)\right)' = f'(x^3) \cdot 3x^2$$

$$\left(f(x^3)\right)' = g(x^3) \cdot 3x^2$$

$$\left(f(x^3)\right)'' = \left(g'(x^3) \cdot 3x^2\right)(3x^2) + g(x^3) \cdot 6x$$

$$= (f(x^3)^2 \cdot 9x^4 + 6x\,g(x^3)$$

$$= 9x^4 f(x^6) + 6x\,g(x^3)$$