Sign up for Khan Academy with coach code 3XDPSR.

Read sections 1.1 and 1.2 in your textbook

HW due Tues (8th per)/Wed (7th per): 1.2 #1-6 all, 15-22 all, 33,34,39,41

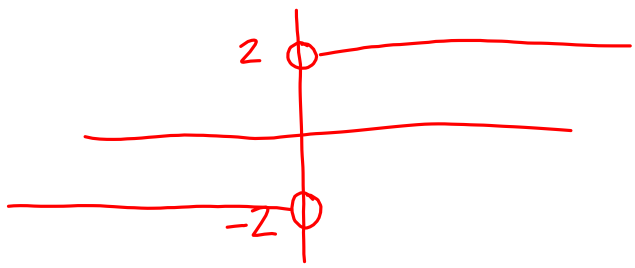
$$\lim_{x \to 0} \frac{|2x|}{x}$$

$$\frac{|2x|}{x} = \begin{cases} \frac{2x}{x} = 2, & 2x > 0 \\ & x > 0 \\ \frac{-(2x)}{x} = -2, & 2x < 0 \\ & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} f(x) = 2$$

$$\lim_{x \to 0^-} f(x) = -2$$

$$\lim_{x \to 0} f(x) = \text{undefined}$$



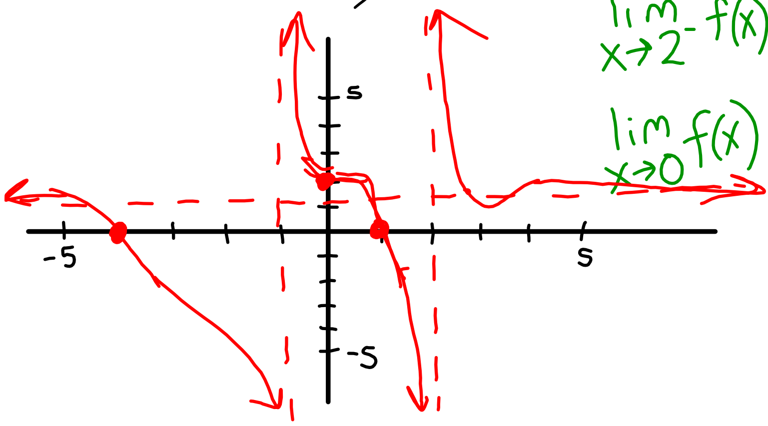
Graph the rational function.

$$f(x) = \frac{(x+4)(x-1)}{(x-2)(x+1)} \approx \frac{x^2}{x^2} = 1$$

$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 0} f(x) = 2$

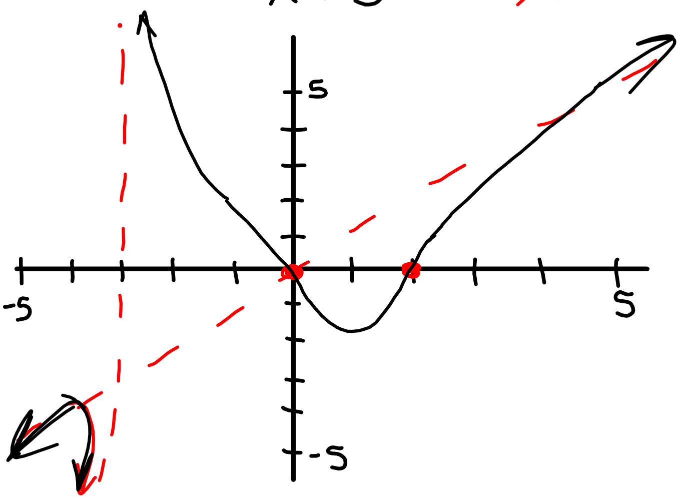


$$f(x) = \frac{x(x-2)}{x+3} \approx \frac{x^2}{x} = x$$

$\lim_{x\to -\infty} f(x) = -\infty$

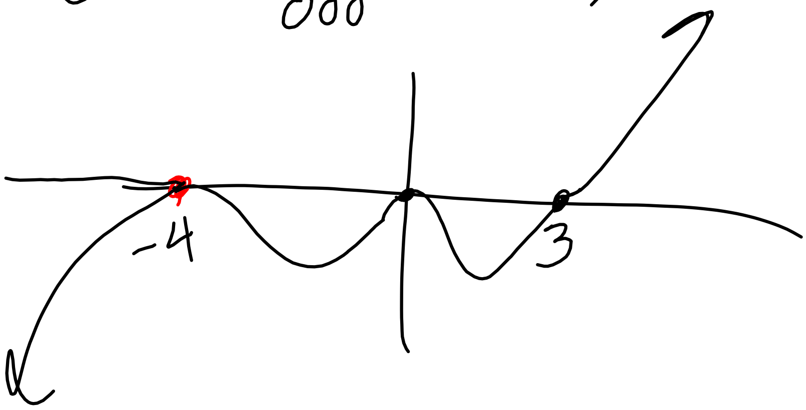
$\lim_{x\to -3^+} f(x) = \infty$

$\lim_{x\to 2} f(x) = 0$



$$y = x^2(x-3)^3(x+4)^6$$

odd



<u>Building up to the $\epsilon - \delta$ Definition of the Limit</u>

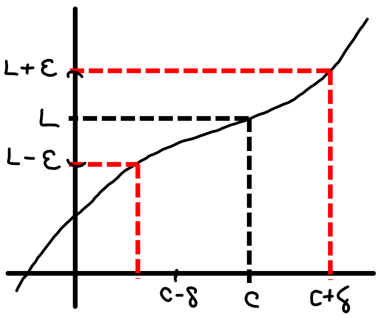
ε = epsilon

δ = delta

<u>Translating the "informal description"</u>: $\lim_{x \to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.

"$f(x)$ becomes arbitrarily close to L"



$f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$0 < |x - c| < \delta$

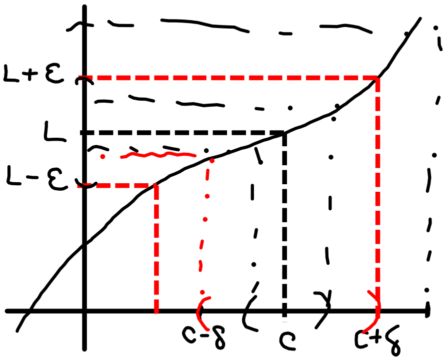
The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.



$\varepsilon - \delta$ Definition of the Limit:

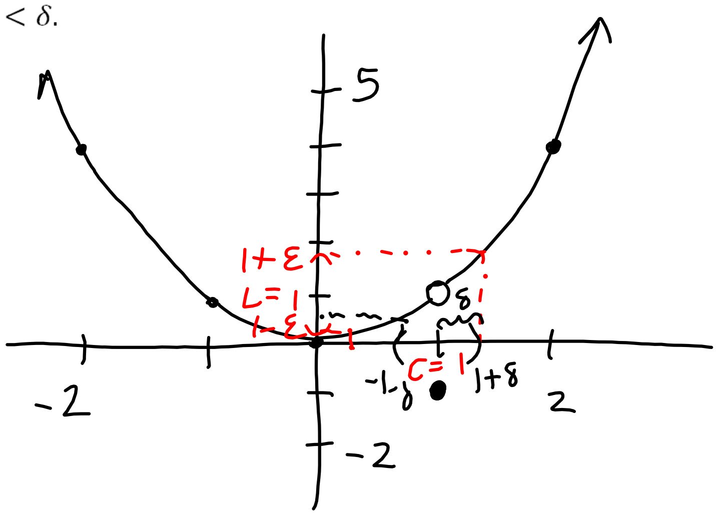
$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = 1$$

$C = 1$ $\quad$ $L = 1$



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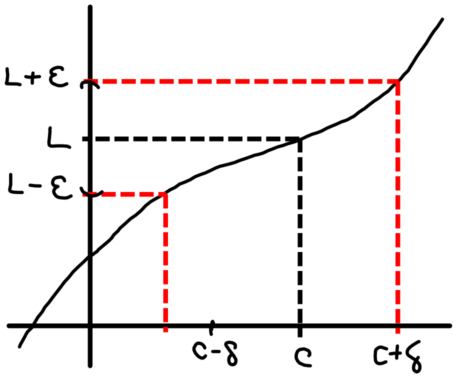

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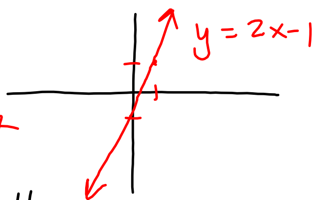

$f(x) = 2x - 1$ $\quad \lim_{x\to 4}(2x-1) = 2(4) - 1 = 7$

Find $\lim_{x\to 4} f(x)$ and prove that is the limit using the ε-δ definition.

$L = 7 \quad c = 4$

Let $\varepsilon > 0$ be given.

$|f(x) - L| = |2x - 1 - 7| = |2x - 8| = 2|x-4| < \varepsilon$

Take $\delta = \varepsilon/2$. so that $|x-4| < \varepsilon/2$

anytime $|x - c| = |x-4| < \varepsilon/2 = \delta$, we have

that $|f(x) - L| = 2|x-4| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$

i.e. $|f(x) - L| < \varepsilon$ and hence

$$\lim_{x\to 4} f(x) = 7.$$