2.1 #1-41 odd; 65-89 odd
2.2 #3-67 odd; 87-95 odd; 97-100 all; 105,106,111,113,115

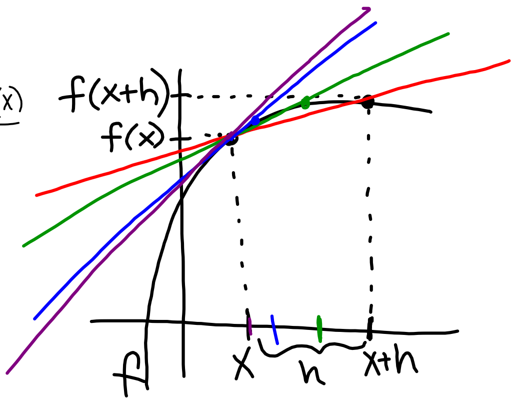
this part due wed 3/22

2.1 The Derivative & The Tangent Line Problem

secant line crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$



what happens as $h \to 0$?

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$f'(x)$	"f prime of x"
$\dfrac{dy}{dx}$	"derivative of y with respect to x"
y'	"y prime"
$\dfrac{d}{dx}[f(x)]$	"the derivative with respect to x of $f(x)$"
$D_x[y]$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

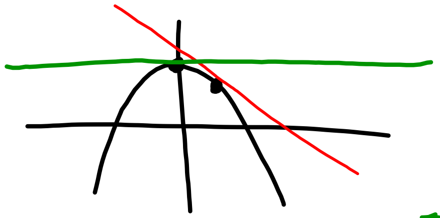
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

8. $g(x) = 5 - x^2$

find slope of tangent line at the points $(2,1)$ & $(0,5)$



$$m = f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$$

@ $(2,1)$: $c = 2$; $f(c) = g(2) = 1$

$f(c+h) = g(2+h) = 5-(2+h)^2$

$$m = \lim_{h\to 0} \frac{5-(2+h)^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{5-(4+4h+h^2)-1}{h}$$

$$= \lim_{h\to 0} \frac{-4h-h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-4-h)}{\cancel{h}}$$

$$= \boxed{-4}$$

@ $(0,5)$ $c = 0$ $g(c) = 5$

$$\lim_{h\to 0} \frac{g(0+h)-g(0)}{h}$$

$$= \lim_{h\to 0} \frac{5-(0+h)^2-5}{h}$$

$$= \lim_{h\to 0} \frac{-h^2}{h^2} =$$

$$= \lim_{h\to 0}(-h) = \boxed{0}$$

20. $f(x) = x^3 + x^2$

find the derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$$

$$= \lim_{h\to 0} \frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{x^2}+2xh+h^2-\cancel{x^3}-\cancel{x^2}}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2+2x+h)}{\cancel{h}} = \boxed{3x^2+2x}$$

$y = mx + b$

Find the equation of the tangent line to

$c \quad f(c)$

$f(x)=x^3-x$ at the point $(2,6)$.

$m = f'(c)$

$= \lim_{h \to 0} \dfrac{f(c+h)-f(c)}{h}$

$$m = \lim_{h \to 0} \frac{(2+h)^3 - (2+h) - 6}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2^3} + 3\cdot 2^2 \cdot h + 3 \cdot 2 \cdot h^2 + h^3 \cancel{-2} - h \cancel{-6}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12 + 6h + h^2 - 1)}{\cancel{h}} = \boxed{11}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - (x+h) - (x^3 - x)}{h}$$

$$= 3x^2 - 1 \Big|_{x=2} = 11$$

$m = 11 \; ; \; (2,6)$

$y - y_1 = m(x - x_1)$

$y - 6 = 11(x-2)$

$y = 11x - 22 + 6$

$$\boxed{y = 11x - 16}$$