Due Wed. 3/22:

2.1 #1-41 odd;

Due Mon. 3/27:

2.1 #65-89 odd
2.2 #3-67 odd;

Due sometime later next week:

2.2
87-95 odd; 97-100 all; 105,106,111,113,115

2.1 The Derivative & The Tangent Line Problem

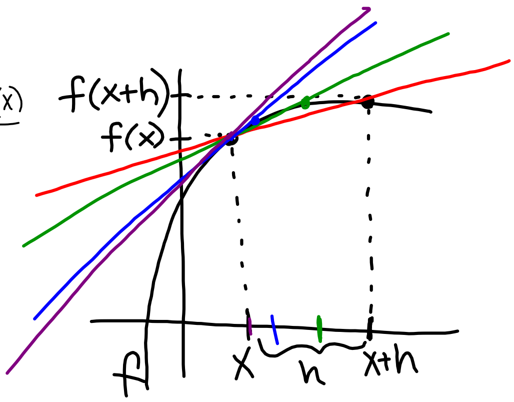
secant line
crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

$$\boxed{f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}}$$

$f'(x)$	"f prime of x"
$\dfrac{dy}{dx}$	"derivative of y with respect to x"
y'	"y prime"
$\dfrac{d}{dx}[f(x)]$	"the derivative with respect to x of $f(x)$"
$D_x[y]$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(c+\Delta x) - f(c)}{\Delta x} = f'(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

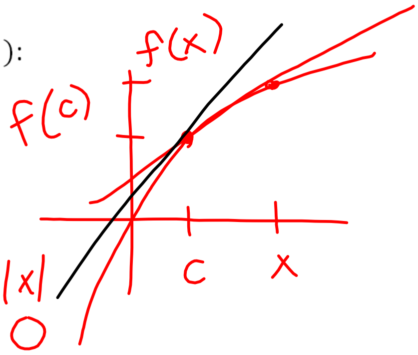
The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

2.1 Differentiability & Continuity

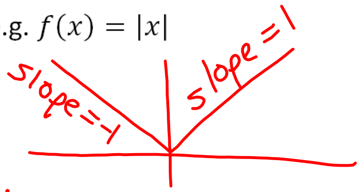
Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$



All differentiable functions are continuous, but not all continuous functions are differentiable.

e.g. $f(x) = |x|$



$$\frac{f(x) - f(c)}{x - c}$$

$f(x) = |x|$

$c = 0$

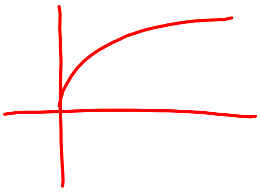
$$\frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1, & x > 0 \\ \frac{-x}{x} = -1, & x < 0 \end{cases}$$

$$\left.\begin{aligned} \lim_{x \to 0^-} f(x) &= -1 \\ \lim_{x \to 0^+} f(x) &= 1 \end{aligned}\right\}$$

Since the left- & right-hand limits are different, the limit in general, and hence the derivative defined by that limit, do not exist.

$f(x) = |x + 3|$

$f(x) = \sqrt{x}$



$$\lim_{x\to 0^+} \frac{\sqrt{x}-\sqrt{0}}{x-0} = \lim_{x\to 0^+} \frac{x^{1/2}}{x^1}$$

$$= \lim_{x\to 0^+} \frac{1}{x^{1/2}}$$

$$\boxed{\frac{a^m}{a^n} = \frac{a^{m-n}}{1} = \frac{1}{a^{n-m}}}$$

$$= \lim_{x\to 0^+} \frac{1}{\sqrt{x}} = \infty$$

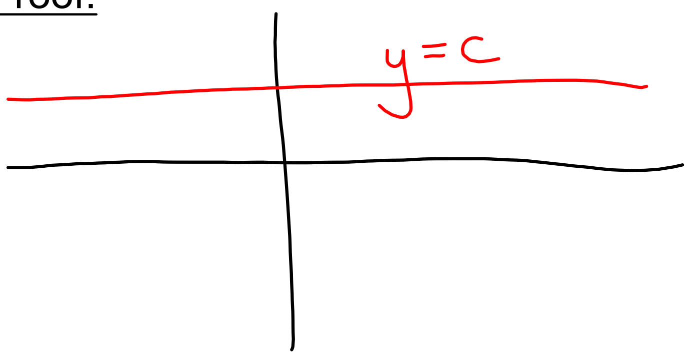
slope of tangent line to $\sqrt{x}$ @ $(0,0)$ is ∞
$\Rightarrow$ vertical tangent line
(undefined slope)

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\quad \frac{d}{dx}[c] = 0$

Proof:



graph of constant function is horizontal line, which has slope = 0

2. Power Rule for $n\epsilon\mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

Special case: $\frac{d}{dx}[x] = 1$

$[x^1]' = 1 \cdot x^0 = 1$

Proof:

Recall the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \frac{n!}{k!\,(n-k)!}a^{n-k}b^k + \cdots + b^n$$

$$[x^n]' = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + \cdots + h^n \cancel{-x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \cdots + h^{n-1}\right)}{\cancel{h}}$$

$\to 0$ $\to 0$

$$= nx^{n-1}$$

Examples:

$\frac{d}{dx}[x^7] = 7x^6$

$\frac{d}{dx}[\pi^3] = 0$

$\frac{d}{dx}[2e] = 0$

$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}(x^{1/2}) = \boxed{\frac{1}{2}x^{-1/2}} = \frac{1}{2x^{1/2}} = \boxed{\frac{1}{2\sqrt{x}}}$

$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}[x^{-3}] = -3x^{-4} = \boxed{\frac{-3}{x^4}}$

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2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

$[5x^2]' = 5 \cdot [x^2]'$

$[3 \sin x]' = 3[\sin x]'$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Examples:

$f(x) = 3x^2$

$f'(x) = 3 \cdot (x^2)' = 3 \cdot 2x = 6x$

$f(x) = \frac{3}{x} = 3x^{-1}$

$f'(x) = -3x^{-2}$

$g(x) = 2x^3 - x^2 + 3x$

$g'(x) = 6x^2 - 2x + 3$

$y = 4x^{3/2} - 5x^4 + 2x^{\frac{1}{3}} - 7$

$y' = 4\left(\frac{3}{2}x^{3/2 - 2/2}\right) - 5(4x^3) + 2\left(\frac{1}{3}x^{\frac{1}{3} - \frac{3}{3}}\right) - 0$

$= 6x^{1/2} - 20x^3 + \frac{2}{3}x^{-2/3}$