Due Mon. 4/10 (8th)

- 2.4 #7-33 odd; 43-89 odd — Chain rule
- 5.1 #41-59 odd; 69,71 — Logarithmic functions

Due Tues 4/11 (8th)/Wed 4/12 (7th)

- 5.4 #33-51 odd; 59, 61 — Exponential functions
- 5.5 #37-69 odd — Log and exp functions with other bases
- 5.6 #39-63 odd — Inverse trig functions

~~TEST~~ Quiz: Wed 4/12 (8th)/ Thurs 4/13 (7th)

Power Rule:
$$\frac{d}{dx}[x^n] = nx^{n-1}$$

d/dx [c]=0

Product Rule:
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Constant Multiple Rule:
$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Quotient Rule:
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Sum & Difference:
$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Chain Rule:
$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

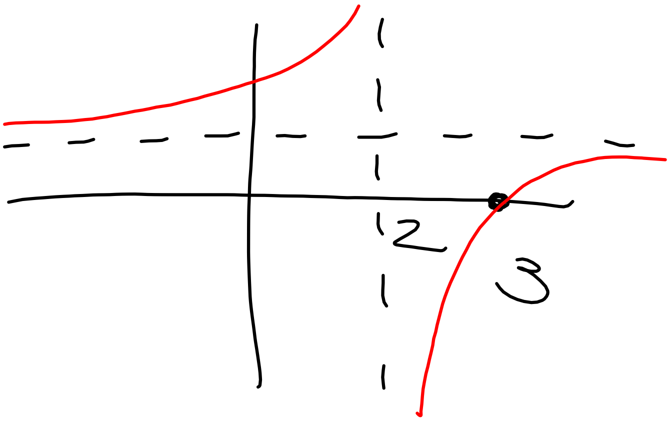
12. $f(x) = \frac{1}{x^5} + \sqrt{x} - \frac{1}{\sqrt[4]{x}}$

$= x^{-5} + x^{1/2} - x^{-1/4}$

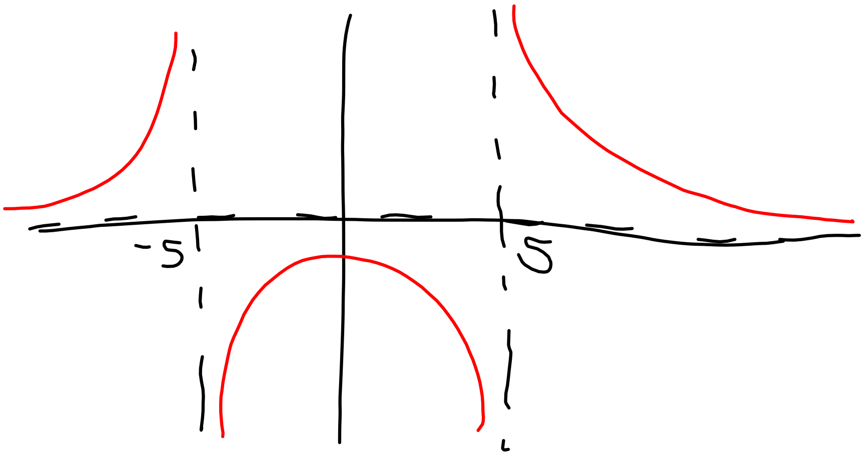
$f'(x) = \boxed{-5x^{-6} + \frac{1}{2}x^{-1/2} + \frac{1}{4}x^{-5/4}}$

$= -\frac{5}{x^6} + \frac{1}{2\sqrt{x}} + \frac{1}{4\sqrt[4]{x^5}}$

22. $\lim_{x \to 2^+} \frac{x-3}{x-2} = \frac{-1}{0^+} = -\infty$




23 $\lim_{x \to 5^-} \frac{1}{(x+5)(x-5)} = -\infty$



Instantaneous rate of change of a function $f(x)$ when $x = c$ is $f'(c)$ <-- slope of tangent line through a single point

Average rate of change of a function $f(x)$ on the interval $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$ <-- slope of secant line through two points

Given a position function $s(t) = gt^2 + v_0 t + s_0$,

Since velocity is the rate of change of position,

The instantaneous velocity at time $t = c$ is $s'(c)$

The average velocity on the interval $[a, b]$ is $\frac{s(b)-s(a)}{b-a}$

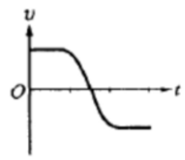
**The Derivative**

The slope of the tangent line to the graph of $f$ at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

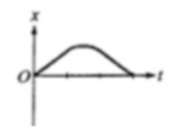
The derivative of f at x is given by

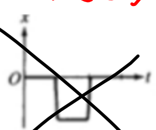
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

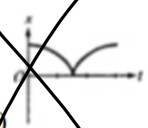


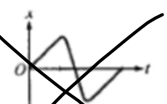
f'(t)

The graph above ~~shows velocity v versus time t for an object in linear motion~~. Which of the following is a possible graph of ~~position x versus time t for this object?~~ f(t)?

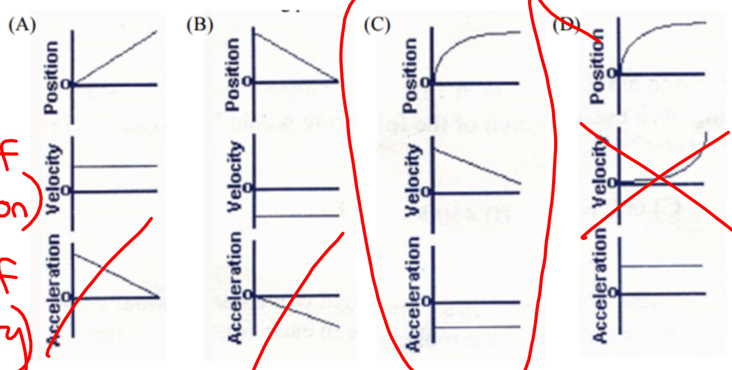
(A) 

(B) 

(C) 

(D) 

Which of the following sets of graphs below might be the corresponding graphs of position, velocity, and acceleration vs time for a moving particle?

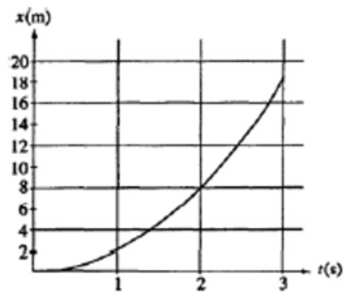



(slope of position)

(slope of velocity)

$$\frac{x(2)-x(1)}{2-1}$$

$$\frac{8-2}{1} = 6$$




The graph above represents position x versus time t for an object being acted on by a constant force. The average speed during the interval between 1 s and 2 s is most nearly

(A) 2 m/s (B) 4 m/s (C) 5 m/s (D) 6 m/s

Consider the motion of an object given by the position vs. time graph shown below. For what time(s) is the speed of the object greatest? where is slope greatest in magnitude ($\pm$)

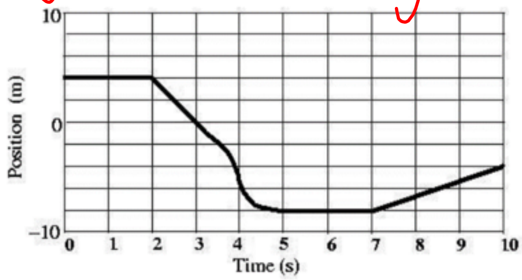
*(A) At all times from t = 0.0 s → t = 2.0 s*

*(B) At time t = 3.0 s*

*(C) At time t = 4.0 s*

*(D) At all times from t = 5.0 s → t = 7.0 s*

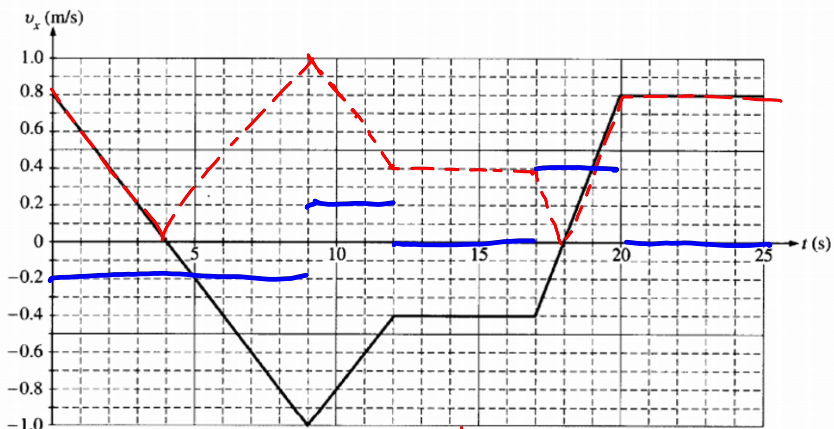
*(E) At time t = 8.5 s*




speed = |velocity|

2000B1 (modified) A 0.50 kg cart moves on a straight horizontal track. The graph of velocity $v$ versus time $t$ for the cart is given below.

red = speed




a. Indicate every time t for which the cart is at rest. @ t = 4 & t = 18

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing. (4,9) ∪ (18,20)

c. ~~Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.~~

d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from t = 0 to t = 25 s.

Find $y', y'', y''', y^{(4)}, y^{(5)}, \ldots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$

$y' = 15x^2 - 6x$

$y'' = 30x - 6$

$y^{(3)} = 30$

$y^{(4)} = 0$

$y^{(5)} = 0$

$y = x^6 + 2x^5 - 3x^4 + 2x - 5$

$y' = 6x^5 + 10x^4 - 12x^3 + 2$

$y'' = 30x^4 + 40x^3 - 36x^2$

$y^{(3)} = 120x^3 + 120x^2 - 72x$

$y^{(4)} = 360x^2 + 240x - 72$

$y^{(5)} = 720x + 240$

$y^{(6)} = 720$

$y^{(7)} = 0$

...

Find $y', y'', y''', y^{(4)}, y^{(5)}, \ldots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$

$y = x^6 + 2x^5 - 3x^4 + 2x - 5$

If $f(x)$ is a polynomial of degree $n$, then $f^{(n+1)}(x) = 0$.

If $f(x) = x^n$, then $f^{(n)}(x) = n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

$f(x) = 3x^9 - 15x^4 + 23x^{16} - 201x^7 - 3$

$f^{(17)} = 0$

$$f(x) = 5\sin\left(3\cos 2x^5\right)$$

$$\left[f(g(x))\right]' = f'(g(x)) \cdot g'(x)$$

$$f'(x) = \boxed{5\cos\left(3\cos 2x^5\right) \cdot \left(-3\sin\left(2x^5\right)\right) \cdot 10x^4}$$

$$= -150x^4 \cos\left(3\cos 2x^5\right)\sin\left(2x^5\right)$$

$$f(x) = x\sin x\sqrt{x-1} = \left[(x)(\sin x)\right]\left((x-1)^{1/2}\right)$$

$$f'(x) = \left[(x)(\sin x)\right]' \cdot \sqrt{x-1} + x\sin x\left[(x-1)^{1/2}\right]'$$

$$= \boxed{\left[1 \cdot \sin x + x\cos x\right]\sqrt{x-1} + x\sin x\left[\frac{1}{2}(x-1)^{-1/2} \cdot 1\right]}$$

$$f(x) = \sec^2(\sin(3x)) = [\sec(\sin(3x))]^2$$

$$f'(x) = [2\sec(\sin(3x))] \cdot [\sec(\sin 3x)\tan(\sin 3x)] \cdot [\cos 3x] \cdot 3$$

$$f(x) = \cos\left(\sqrt{\tan^2 x - 2x}\right)$$

$$= \cos\left[\left([\tan x]^2 - 2x\right)^{1/2}\right]$$

$$f'(x) = \left[-\sin\sqrt{\tan^2 x - 2x}\right] \cdot \frac{1}{2}(\tan^2 x - 2x)^{-1/2} \cdot [2 \cdot \tan x \cdot \sec^2 x - 2]$$

$$f(x) = \cot(5x^2-3x)$$

$$f'(x) = \boxed{\left[-\csc^2(5x^2-3x)\right]\cdot(10x-3)}$$

$$f(x) = \sqrt[3]{\csc(4x)} = \left[\csc(4x)\right]^{1/3}$$

$$f'(x) = \frac{1}{3}\left[\csc(4x)\right]^{-2/3}\cdot\left[-\csc(4x)\cot(4x)\right]\cdot 4$$

$$f(x) = \frac{\sin 2x}{x^3} = (x^{-3})(\sin 2x)$$

using quotient rule:

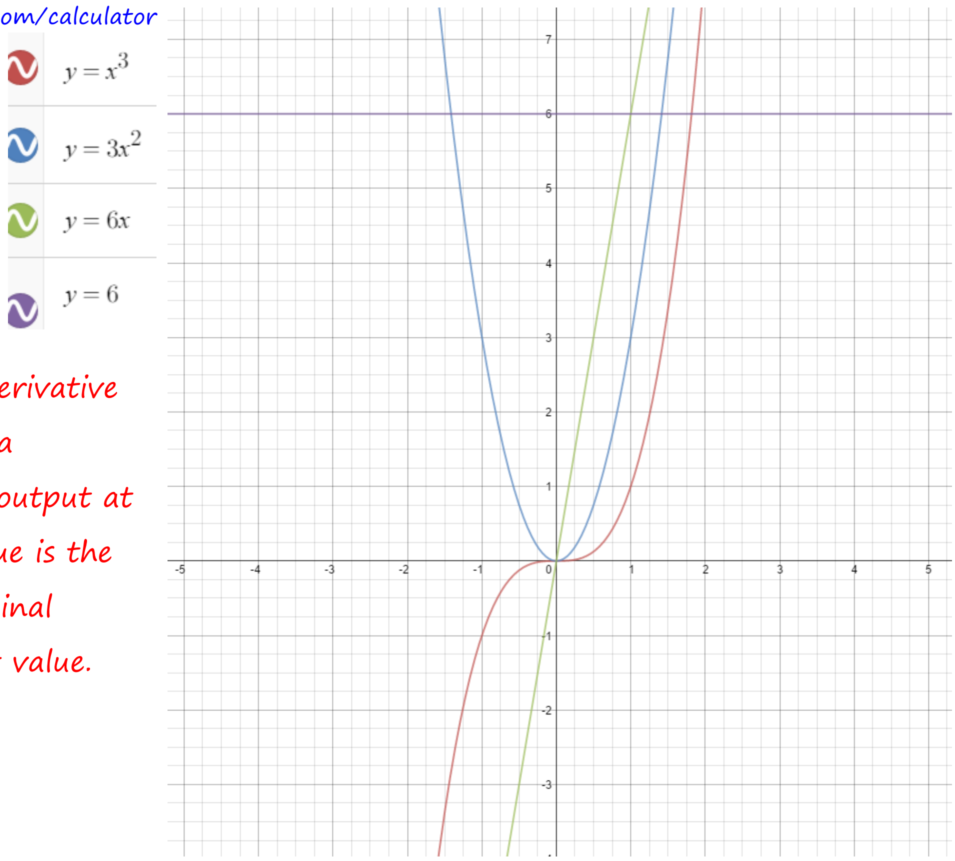
$$f'(x) = \frac{x^3(\cos 2x)(2) - (\sin 2x)(3x^2)}{(x^3)^2}$$

using product rule:

$$f'(x) = (x^{-3})'(\sin 2x) + (x^{-3})(\sin 2x)'$$

$$f'(x) = (-3x^{-4})(\sin 2x) + (x^{-3})(\cos 2x)(2)$$

https://www.desmos.com/calculator




Note that the derivative of a function is a function whose output at a particular value is the slope of the original function at that value.

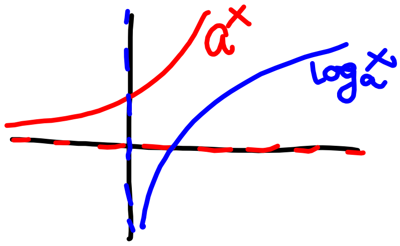
Ch 5 - Derivatives of Logarithmic and Exponential Functions

recall: $\ln x = \log_e x$

$e \approx 2.7$

$\log_2 8 = 3 \iff 2^3 = 8$

$\log_a b = c \iff a^c = b$




$y = 2^x$

$x$ = the power to which we raise 2 to get $y$

= the # of times we multiply 2 by itself to get $y$

$= \log_2 y$

$$\frac{d}{dx}\left[2^x\right] = 2^x \ln 2$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{x \ln 2}$$

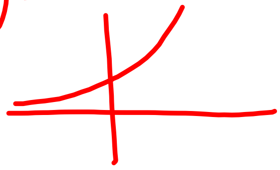
$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u \cdot \ln a}$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{f(x) \cdot \ln a} \cdot f'(x) = \frac{f'(x)}{f(x) \cdot \ln a}$$

$$[e^x]' = e^x \cdot \ln e = e^x \log_e e = e^x$$

$$[e^x]' = e^x$$



$$[\ln x]' = \frac{1}{x \ln e} = \frac{1}{x}$$

Since the derivative of $e^x$ is itself, this means that graphically, at every x-value, the slope of the tangent line at that point is exactly the y-coordinate.

$$f(x) = \ln[\sin(5x^3+2x)]$$

$$f'(x) = \frac{1}{\sin(5x^3+2x)} \cdot \cos(5x^3+2x) \cdot (15x^2+2)$$

$$= (15x^2+2)\cot(5x^3+2x)$$

$$f(x) = (\sec x)(5^{\sin x})$$

$$f'(x) = (\sec x)(5^{\sin x})(\ln 5)(\cos x) + (\sec x \tan x)(5^{\sin x})$$

$$\left[5^{f(x)}\right]' = 5^{f(x)} \cdot \ln 5 \cdot f'(x)$$