Applications of Derivatives

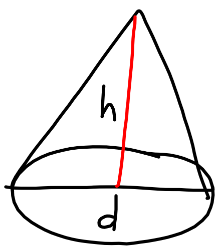
Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates

Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)

Next week:
3.1 # 17-35 odd - Absolute Extrema on an Interval
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

17. $10\ \text{ft}^3/\text{min} = \frac{dV}{dt}$ $\quad d = 3h$
$2r = 3h$

$\frac{dh}{dt} = ?$ when $h = 15$ ft $\quad r = \frac{3h}{2}$

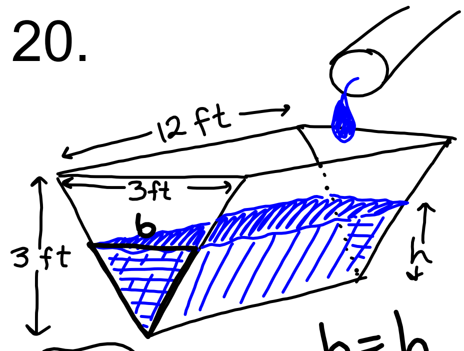


$$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(\frac{3h}{2}\right)^2 \cdot h$$

$$V = \frac{\pi}{3} \cdot \frac{9h^2}{4} \cdot h$$

$$V = \frac{3\pi}{4} h^3 \quad \ldots$$

20.



(a) $\frac{2\,ft^3}{min} = \frac{dV}{dt}$; $\frac{dh}{dt} = ?$ when $h = 1$ ft

$b = h$

$V = (\text{area of } \Delta) \cdot \text{height}$

$V = \frac{1}{2} bh \cdot 12$

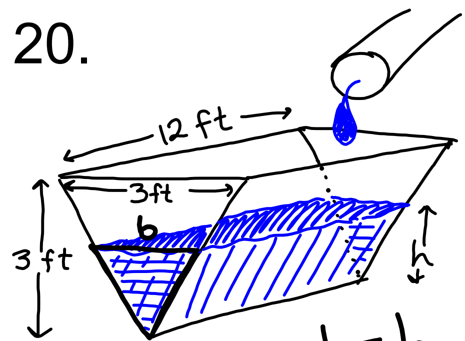
$V = 6h^2$

$\frac{dV}{dt} = 12h \cdot \frac{dh}{dt}$

$V = (\text{area of base}) \cdot \text{height}$

$$\boxed{\frac{1}{6}\,\frac{ft}{min}} = \frac{2\,ft^3/min}{12ft \cdot 1ft} = \frac{\frac{dV}{dt}}{12h} = \frac{dh}{dt}$$

20.



(b) $\frac{3}{8}\,\frac{in}{min} = \frac{dh}{dt}$ when $h = 2$ ft

$\frac{dV}{dt} = ?$

$b = h$

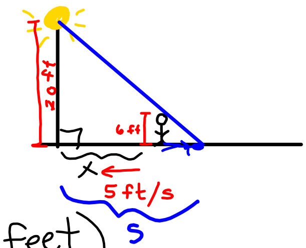
$\frac{dV}{dt} = 12h \cdot \frac{dh}{dt}$

$= \cancel{12}ft \cdot \cancel{2}ft \cdot \frac{3\,\cancel{in}}{\cancel{8}_{4}\,min} \cdot \frac{1ft}{\cancel{12}\,\cancel{in}}$

$$\frac{dV}{dt} = \boxed{\frac{3}{4}\,ft^3/min}$$

30. A man 6 ft tall walks toward a light that is 20 ft above the ground at a rate of 5 ft/s. When he is 10 ft from the base of the light,

(a) at what rate is the tip of his shadow moving?



Let x = the distance between the man & the light (in feet)

Let s = the distance between the light & tip of shadow (in feet)

$\frac{ds}{dt} = ?$ when $x = 10$ ft ; $\frac{dx}{dt} = -5$ ft/s

$\frac{20 \cancel{ft}}{6 \cancel{ft}} = \frac{s}{s-x}$ OR $\frac{20}{s} = \frac{6}{s-x}$

$\frac{10}{3} = \frac{s}{s-x}$

$10(s-x) = 3s$

$10s - 10x = 3s$

$7s = 10x$

$7 \cdot \frac{ds}{dt} = 10 \frac{dx}{dt} \rightarrow$

$\frac{ds}{dt} = \frac{10}{7} \cdot (-5)$

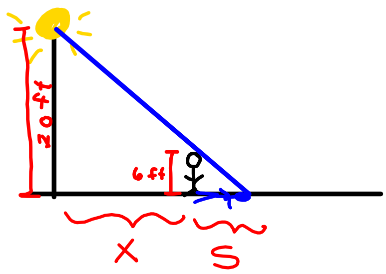
$= \boxed{\frac{-50}{7} \frac{ft}{s}}$

36. A man 6 ft tall walks toward a light that is 20 ft above the ground at a rate of 5 ft/s. When he is 10 ft from the base of the light,

Let x = dist. betw man & light

Let s = length of shadow

(b) at what rate is the length of his shadow changing?



$\frac{dx}{dt} = -5$ ft/min ; $\frac{ds}{dt} = ?$

$\frac{20}{x+s} = \frac{6}{s} \Rightarrow 20s = 6(x+s)$

$20s = 6x + 6s$

$14s = 6x$

$s = \frac{3}{7}x$

$\frac{ds}{dt} = \frac{3}{7} \cdot \frac{dx}{dt}$

$= \frac{3}{7}(-5) = \boxed{\frac{-15}{7} \text{ ft/s}}$