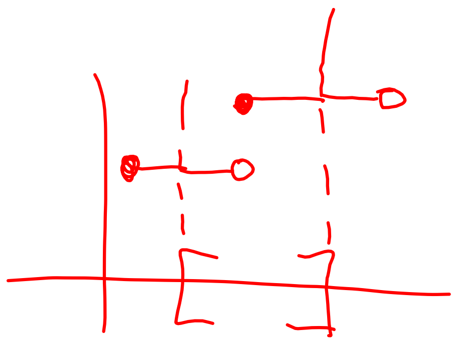
Applications of Derivatives

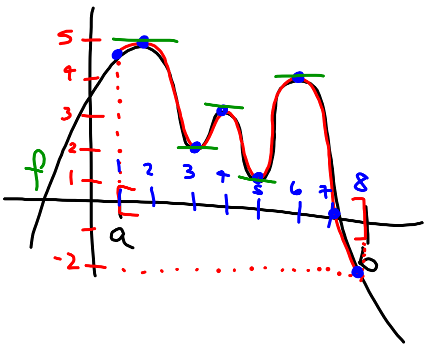
Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates
Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)
Due Monday 5/8:
3.1 # 17-35 odd - Absolute Extrema on an Interval

Due Tues (8th per) / Wed (7th per):
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
Due Fri 5/12:
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity



3.1 Extrema on an Interval

- maxima & minima
- relative & absolute



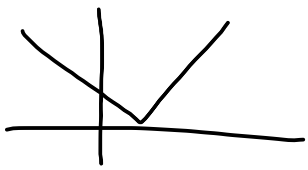
relative minima:
(3,2), (5,1)

relative maxima:
(2,5), (4,3), (6,4)

absolute maximum:
5 @ (2,5)

absolute minimum:
-2 @ (8,-2)

$f(x)$ can have a relative maximum or minimum when $f'(x) = 0$. or



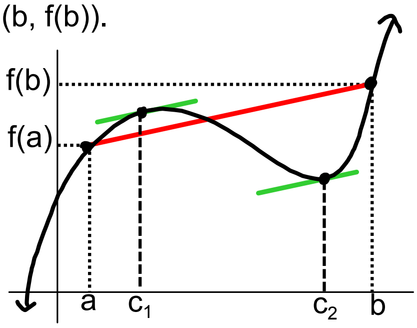
$f'(x)$ is undefined.

We call such x-values

Critical #'s of f.

3.2 Rolle's Theorem & The Mean Value Theorem

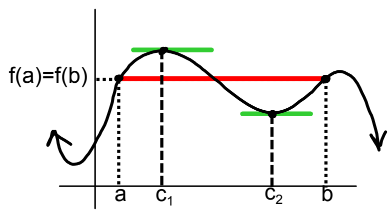
The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



If f is continuous on [a,b] & differentiable on (a,b), then $\exists\ c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



Steps to solve MVT problems:
1. Is f continuous on [a,b]?
2. Is f differentiable on (a,b)?
3. If Rolle's Theorem, is f(a)=f(b)? ⟸
4. Find (f(b)-f(a))/(b-a)
5. Find f'(x)
6. Set #~~3~~4&~~4~~5 equal, solve for x
7. Solution is the values of x from #5 that lie in (a,b)

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do *f′* and *f″* tell us about *f*?

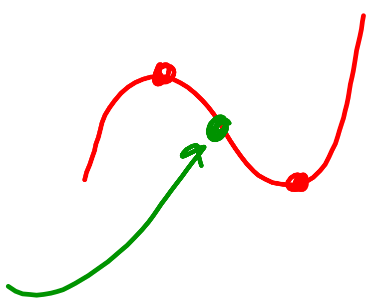
Recall that *f′* is the rate of change or slope of *f*,
f″ is the slope or rate of change of *f′*.

f'	f
+	increasing
−	decreasing

f''	f'	f
+	increasing	concave up
−	decreasing	concave down

f′(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f′(x) is undefined) are called <u>critical numbers</u>.

f″(x)=0 when f changes concavity.
The points where concavity changes are called <u>inflection points</u>.



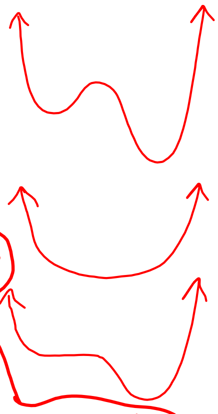
$f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

critical #'s: 0, 3

$f'(-1)$, $f'(1)$, $f'(4)$

$- \quad 0 \quad - \quad 3 \quad +$



f is decreasing on $(-\infty, 3)$

f is increasing on $(3, \infty)$

f has a relative (& absolute) minimum @ $(3, -27)$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

$f''(-1)$, $f''(1)$, $f''(3)$

$+ \quad 0 \quad - \quad 2 \quad +$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

f is concave down on $(0, 2)$

f has inflection points @ $(0, 0)$ & $(2, -16)$