Applications of Derivatives

Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3

2.6 # 11-17 odd - Related Rates

Due Friday, 5/5:

2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)

Due Monday 5/8:

3.1 # 17-35 odd - Absolute Extrema on an Interval

Due Tues (8th per) / Wed (7th per):

3.2 # 9-21 odd - Rolle's Theorem

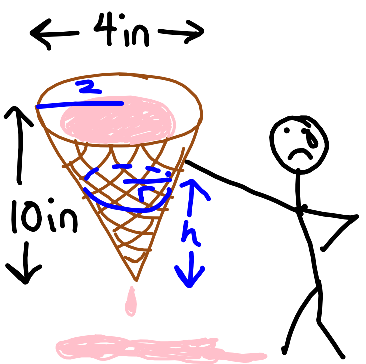
3.2 # 33-45 odd - Mean Value Theorem

Due Fri 5/12:

3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema

3.4 #15-39 odd - Inflection Points and Concavity

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$\frac{dh}{dt} = \frac{-1\text{in}}{5\text{min}}$; $\frac{dV}{dt} = ?\ \frac{\text{in}^3}{\text{min}}$ when $h = 5\text{in}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{2}{10}$

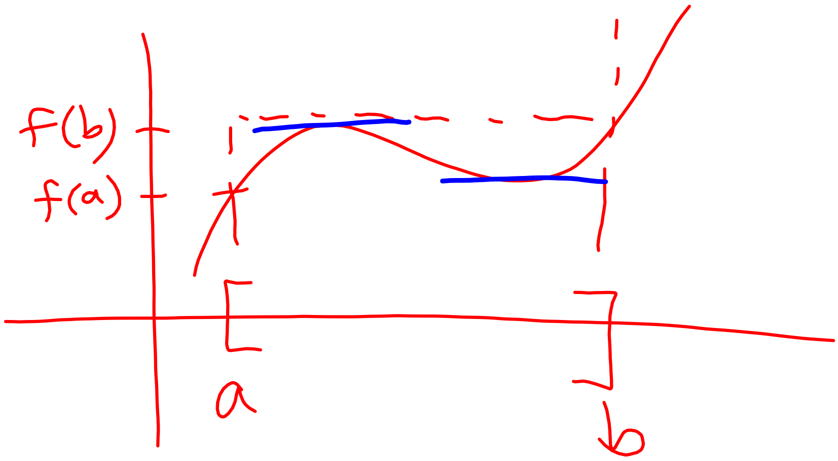
$r = \frac{h}{5}$

$V = \frac{1}{3}\pi\left(\frac{h}{5}\right)^2 \cdot h$

$V = \frac{\pi}{3 \cdot 25} h^3$

$\frac{dV}{dt} = \frac{\pi}{25} h^2 \cdot \frac{dh}{dt}$

$= \frac{\pi}{25}(5)^2 \cdot \left(\frac{-1}{5}\right) = \boxed{\frac{-\pi}{5}\ \text{in}^3/\text{min}}$



3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do f' and f'' tell us about f?

Recall that f' is the rate of change or slope of f,
f'' is the slope or rate of change of f'.

f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down

30. $f(x) = \dfrac{x+3}{x^2}$

vertical asymptote @ $x = 0$
$(-3,0)$ is an x-intercept / zero
domain: $(-\infty,0)\cup(0,\infty)$
horizontal asymptote @ $y = 0$

$$f'(x) = \frac{d}{dx}\left((x+3)/x^{\wedge}2\right) = \frac{-(x+6)}{x^3}$$

$$= \frac{x^2(1) - (x+3)\cdot 2x}{(x^2)^2} = \frac{\cancel{x}(x - 2x - 6)}{x^{\cancel{4}3}} = \frac{-x-6}{x^3}$$

critical #'s: $-6, 0$

$f'(-7)$, $f'(-1)$, $f'(1)$

$-$ | -6 | $+$ | 0 | $-$

f is decreasing on $(-\infty,-6)\cup(0,\infty)$
f is increasing on $(-6,0)$
f has a relative minimum @ $\left(-6, \frac{-1}{12}\right)$
f has no relative maxima

$$f''(x) = \frac{d^2}{dx^2}\left(f(x)\right)$$

$$= \frac{x^3(-1) + (+x+6)(3x^2)}{(x^3)^2}$$

$$= \frac{x^2(-x+3x+18)}{x^6} = \frac{2x+18}{x^4} = \frac{2(x+9)}{x^4}$$

$f''(-10)$, $f''(-1)$, $f''(1)$

$-$ | -9 | $+$ | 0 | $+$

f is concave down on $(-\infty,-9)$
f is concave up on $(-9,0)\cup(0,\infty)$

f has an inflection point @ $\left(-9, \frac{-2}{27}\right)$