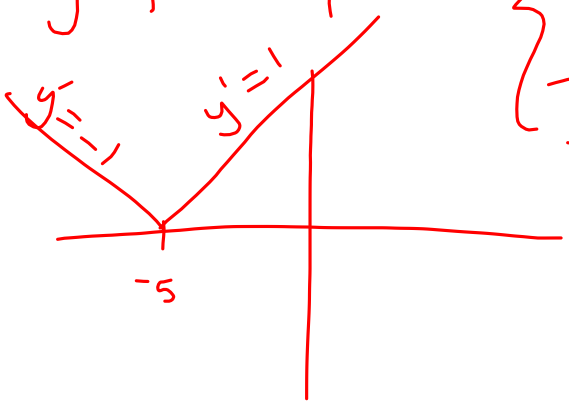
Applications of Derivatives
Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates
Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)
Due Monday 5/8:
3.1 # 17-35 odd - Absolute Extrema on an Interval
Due Tues (8th per) / Wed (7th per):
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem

Due Fri 5/12:
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

$$y = |x+5| = \begin{cases} x+5, & x \geq -5 \\ -(x+5), & x < -5 \end{cases}$$

$-x-5$



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

4. $f(x) = (x+3)^2(x-1)$

a. Determine if Rolle's Theorem can be applied on the closed interval $[-3,1]$, and if so, find all values of c in $(-3,1)$ guaranteed by that theorem such that $f'(c) = 0$.

b. Determine if the Mean Value Theorem can be applied on the closed interval $[-2,2]$, and if so, find all values of c in $(-2,2)$ guaranteed by that theorem such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

f is a polynomial & hence it is continuous & differentiable on $(-\infty,\infty)$ $\rightarrow$ MVT applies

$f(-3) = 0 = f(1)$ $\Rightarrow$ Rolle's Theorem applies

(a) $f'(x) = \frac{d}{dx}\left(((x+3)\wedge 2)\cdot(x-1)\right) =$

$= \left[(x^2+6x+9)(x-1)\right]' = \left[x^3+6x^2+9x-x^2-6x-9\right]' =$

$= \left[x^3+5x^2+3x-9\right]' = 3x^2+10x+3$

$0 = (3x+1)(x+3)$ $\boxed{x = -1/3}$, $\cancel{-3}$

(b) $\frac{f(2)-f(-2)}{2-(-2)} = \frac{(2+3)^2(2-1)-(-2+3)^2(-2-1)}{4} =$

$= \frac{25+3}{4} = \frac{28}{4} = 7$

$3x^2+10x+3 = 7$

solve$(3x\wedge 2+10x+3=7, x)$

$3x^2+10x-4=0$

$x = \frac{-10 \pm \sqrt{10^2-4(3)(-4)}}{2(3)} = \frac{\pm\sqrt{37}+5}{3}$

$\boxed{x = \frac{5-\sqrt{37}}{3}}$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do *f'* and *f''* tell us about *f*?

Recall that *f'* is the rate of change or slope of *f*,
f'' is the slope or rate of change of *f'*.

f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down

$$f(x) = \frac{x+1}{\sqrt{x}} \qquad \text{domain: } (0, \infty)$$

$$f'(x) = \frac{x^{1/2}(1) - (x+1)\cdot\frac{1}{2}x^{-1/2}}{(\sqrt{x})^2} = \frac{x^{1/2} - (x+1)\cdot\frac{1}{2}x^{-1/2}}{x} \cdot \frac{2x^{1/2}}{2x^{1/2}}$$

$$f'(x) = \frac{2x-(x+1)}{2x^{3/2}} = \boxed{\frac{x-1}{2x^{3/2}} = f'(x)}$$

critical #'s: 0, 1

$$f''(x) = \frac{(2x^{3/2})(1) - (x-1)(3x^{1/2})}{(2x^{3/2})^2} = \frac{x^{1/2}(2x-3x+3)}{4x^3} =$$

$$f''(x) = \frac{-x+3}{4x^{3-1/2}} = \boxed{\frac{-(x-3)}{4x^{5/2}} = f''(x)}$$

#'s: 0, 3

To find ↗, ↘, max, & min:

$\cancel{f'(-1)}$, $f'(\frac{1}{2})$, $f'(2)$

0 — 1 +

↘ ↗

f is decreasing on $(0,1)$
& increasing on $(1,\infty)$
f has a relative minimum @ $(1,2)$

To find ∪ ∩ & inflection pts:

$f''(1)$, $f''(4)$

0 + 3 —

∪ ∩

f is concave up on $(0,3)$ &
concave down on $(3,\infty)$
f has an inflection point @ $\left(3, \frac{4}{\sqrt{3}}\right)$