Applications of Derivatives

Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates
Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)
Due Monday 5/8:
3.1 # 17-35 odd - Absolute Extrema on an Interval
Due Tues (8th per) / Wed (7th per):
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
Due Fri 5/12:
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity
Due Mon 5/15:
3.5 #15-31 odd - Limits at Infinity
8.7 #15-31 odd - l'Hopital's Rule
Due Fri 5/19:
3.7 #3,5,17,19,23 - Optimization

$$\left(e^{x^2}\right)' = e^{x^2} \cdot 2x$$

$$\left(e^2\right)^x = e^{2x}$$

$$e^{2^x} = e^{2^x} \cdot 2^x \ln 2$$

$$y = x^x$$

40. $\lim_{x\to\infty} x\tan\frac{1}{x}$

$\infty \cdot 0 = \frac{\infty}{\left(\frac{1}{0}\right)} = \frac{0}{\left(\frac{1}{\infty}\right)}$

$\lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}}$ $\frac{0}{0}$

$= \lim_{x\to\infty} \frac{\sec^2\frac{1}{x} \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$

$= \boxed{1}$

$\lim_{x\to\infty} \frac{x}{\cot\frac{1}{x}}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{1}{-\csc^2\frac{1}{x} \cdot \frac{-1}{x^2}}$

$= \lim_{x\to\infty} \frac{x^2}{\csc^2\frac{1}{x}}$

2. $x^3 + y^2 = 10$

a. Find y' in terms of x and y.

b. Find y'' in terms of x and y.

$y' = \frac{dy}{dx}$

(a) $\frac{d}{dx}[x^3+y^2] = \frac{d}{dx}[10]$

$3x^2 + 2y\cdot y' = 0$

$2yy' = -3x^2$

$\boxed{y' = \frac{-3x^2}{2y}}$

$y'' = \frac{d}{dx}\left[\frac{-3x^2}{2y}\right]$

$= \frac{(2y)(-6x) - (-3x^2)(2y')}{(2y)^2}$

$y'' = \frac{-12xy + 6x^2\left(\frac{-3x^2}{2y}\right)}{4y^2}$

3. $f(x) = (x-3)^2(x+2)$

a. Find the critical numbers of the function.

b. Find the absolute maximum and absolute minimum (if any) on the closed interval $[-1,1]$.

c. Find all open intervals on the function's domain on which it is increasing or decreasing.

(a) find $\frac{d}{dx}(((x-3)\wedge 2)\cdot(x+2)) = 3x^2-8x-3$

solve $(3x^2-8x-3=0, x)$

$f(x) = (x^2-6x+9)(x+2) = x^3-6x^2+9x+2x^2-12x+18$

$= x^3-4x^2-3x+18$

$f'(x) = 3x^2-8x-3$

$(3x+1)(x-3) = 0$

$\boxed{x = -1/3,\ x = 3}$

(b) $f(-1) = 16$

$f(-1/3) = \frac{500}{27} \approx 18.5$ abs max

$f(1) = 12$ abs min

(c) $f'(-1)$ $-1/3$ $f'(0)$ 3 $f'(4)$

$+$ $-$ $+$

rel max @ $(-1/3, f(-1/3))$

rel min @ $(3, f(-3))$

f is increasing on $(-\infty, -1/3) \cup (3, \infty)$

f is decreasing on $(-1/3, 3)$

2. The volume v of a sphere with respect to its radius r is given by $v = \frac{4}{3}\pi r^3$.

a. Find the instantaneous rate of change of v with respect to r when $r = 3cm$.

b. Find the average rate of change of v as r changes from 1 cm to 2 cm.

(a) $\frac{d}{dr}[V] = \frac{d}{dr}\left[\frac{4}{3}\pi r^3\right]$

$\frac{dV}{dr} = 4\pi r^2\Big|_{r=3\,cm} = 4\pi(3)^2 = \boxed{36\pi\ cm^2}$

(b) $\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{2-1} = \boxed{\frac{28\pi}{3}\ cm^2}$

(c) How fast is the radius shrinking when r = 1cm if the volume is shrinking at 1 cm³/s ?

$V = \frac{4}{3}\pi r^3$

$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2} = \frac{-1}{4\pi(1)^2} = \boxed{\frac{-1}{4\pi}\ cm/s}$

18. Find the limit (if it exists).

$$\lim_{x\to 4^+}\frac{|x-4|}{x-4} = \lim_{x\to 4^+}\begin{cases}1, & x>4\\ -1, & x<4\end{cases} = \boxed{1}$$

19. Find the limit (if it exists).

$$\lim_{x\to 0^+} f(x),\quad f(x)=\begin{cases}2x^2+2x+1, & x\le 0\\ x-3, & x>0\end{cases} = \boxed{-3}$$

20. Find the limit (if it exists).

$$\lim_{x\to 2} f(x),\quad f(x)=\begin{cases}10-x, & x\le 2\\ x^2+2x, & x>2\end{cases} = \boxed{8}$$

Mean Value Theorem:

If f is continuous on $[a,b]$ & differentiable on (a,b), then $\exists\, c\in(a,b)$ s.t. $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

Does MVT apply & if so, find the c.

$f(x)=\dfrac{1}{x-2}$, $[0,3]$

MVT does not apply

$f(x)=e^x$, $[1,2]$

$f'(x)=e^x$ $\qquad \dfrac{f(2)-f(1)}{2-1}=\dfrac{e^2-e}{1}$

solve$(e^x=e^2-e, x)$

$\ln(e^x)=\ln(e^2-e)$

$x=\ln(e^2-e)=1+\ln(e-1)$