Applications of Derivatives

Due: 8th period - Tuesday, 5/2
7th period - Wednesday, 5/3
2.6 # 11-17 odd - Related Rates
Due Friday, 5/5:
2.6 # 19-27 odd, 35 - Related Rates (more challenging problems)
Due Monday 5/8:
3.1 # 17-35 odd - Absolute Extrema on an Interval
Due Tues (8th per) / Wed (7th per):
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
Due Fri 5/12:
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity
Due Mon 5/15:
3.5 #15-31 odd - Limits at Infinity
8.7 #15-31 odd - l'Hopital's Rule
Due Fri 5/19:
3.7 #3,5,17,19 - Optimization

3.7 Optimization Problems

x, y

4. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

$xy = 192$

$x = \frac{192}{y}$

$S(x,y) = x + 3y$

$S(y) = \frac{192}{y} + 3y = 192y^{-1} + 3y$

$S'(y) = -192y^{-2} + 3$

$S'(y) = \frac{-192}{y^2} + 3$

$\frac{-192}{y^2} + 3 = 0$

$\cancel{y^2} \cdot \frac{-192}{\cancel{y^2}} = -3 \cdot y^2$

$-192 = -3y^2$

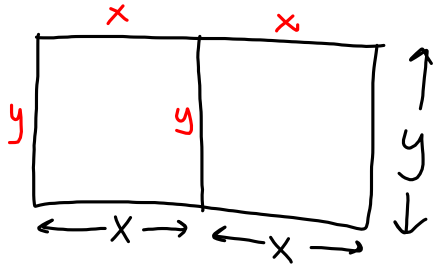
$64 = y^2$

$y = \pm 8$

$y = 8$

$x = \frac{192}{8} = 24$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



$4x + 3y = 200 \rightarrow 4x = 200 - 3y$

$x = 50 - \frac{3}{4}y$

$A(x,y) = 2xy$

$A(y) = 2\left(50 - \frac{3}{4}y\right)y$

$A(y) = 100y - \frac{3}{2}y^2$

$A'(y) = 100 - 3y$

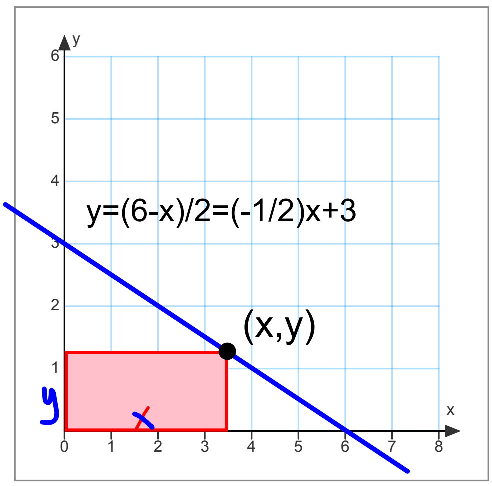
$100 - 3y = 0$

$100 = 3y$

$\boxed{y = \frac{100}{3}\text{ ft}}$

$x = 50 - \frac{\cancel{3}}{\cancel{4}}\left(\frac{\overset{25}{\cancel{100}}}{\cancel{3}}\right) = \boxed{25\text{ ft}}$

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$A(x,y) = xy = x\left(-\frac{1}{2}x + 3\right)$

$A(x) = -\frac{1}{2}x^2 + 3x$

$A'(x) = -x + 3 = 0$

$\boxed{x = 3}$

$y = -\frac{1}{2}(3) + 3 = \boxed{\frac{3}{2}}$

44. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ $\qquad 1^\infty$

$\log a^{p} = p\log a$

$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$\ln y = \ln \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$\ln y = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^{x}$

$\ln y = \lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right)$ $\qquad \infty\cdot 0$

$\ln y = \lim_{x\to\infty} \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ $\qquad \frac{0}{0}$

$\ln y = \lim_{x\to\infty} \dfrac{\frac{1}{1+\frac{1}{x}}\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$

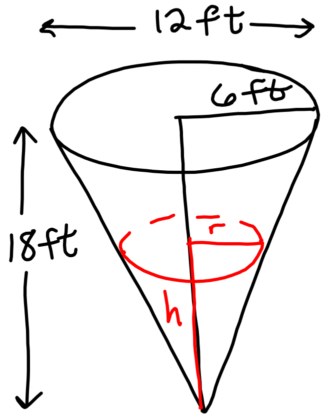
$\ln y = \lim_{x\to\infty} \dfrac{1}{1+\frac{1}{x}} = 1 \Rightarrow \ln y = 1$

$e^{\ln y} = e^1$

$y = \boxed{e}$

$a^{\log_a x} = x$

$\log_a(a^x) = x$



$\frac{dV}{dt} = 18\ ft^3/min$; $\frac{dh}{dt} = ?$ when $h = 10ft$

$\frac{r}{h} = \frac{6}{18}$

$r = h/3$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\cdot\frac{dh}{dt}$

$\frac{dh}{dt} = \dfrac{\frac{dV}{dt}}{\frac{\pi}{9}h^2}$

$\frac{81}{50\pi} = \frac{18\cdot 9}{\pi\, 10^2} = \frac{\cancel{2}\cdot 9\cdot 9}{\pi\cdot \cancel{100}\ 50}$

avg. rate of change of a function $f(x)$
- occurs over an interval $[a, b]$
- never involves derivative
- slope of secant line thru end points

$$\frac{f(b)-f(a)}{b-a}$$

instantaneous rate of change
- occurs @ single value c
- slope of tangent line @ c

$$f'(c)$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$$

what is the average velocity? on $[1, 2]$

translated: what is the average rate of change of position?

$$\frac{s(2)-s(1)}{2-1}$$

average acceleration?
(r.o.c. of velocity)

$$\frac{v(2)-v(1)}{2-1}$$

$f(x)=\sqrt{2x^2-7}$ $=(2x^2-7)^{1/2}$ eq. of tangent line @ $(2,1)$

$y=mx+b$

$m=f'(2)$

$y-y_1=m(x-x_1)$

$y-1=4(x-2)$

$f'(x)=\frac{1}{2}(2x^2-7)^{-1/2}\cdot 4x$

$y=4x-8+1$

$f'(2)=\dfrac{4\cdot 2}{2\sqrt{2\cdot 2^2-7}}=\dfrac{4}{1}$ $4=m$

$\boxed{y=4x-7}$