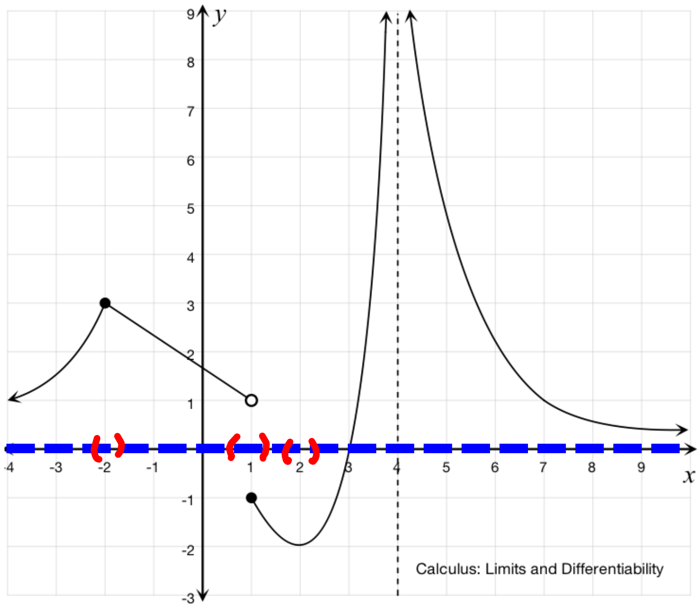
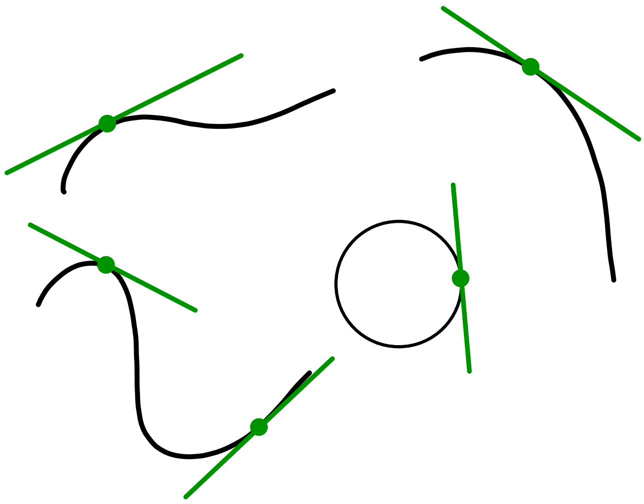
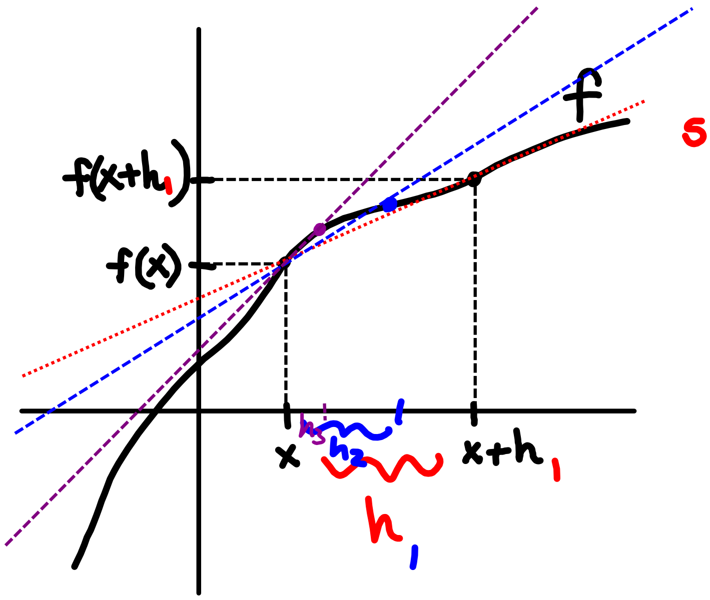


as x approaches..	$f(x)$ approaches...
2	-2
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines





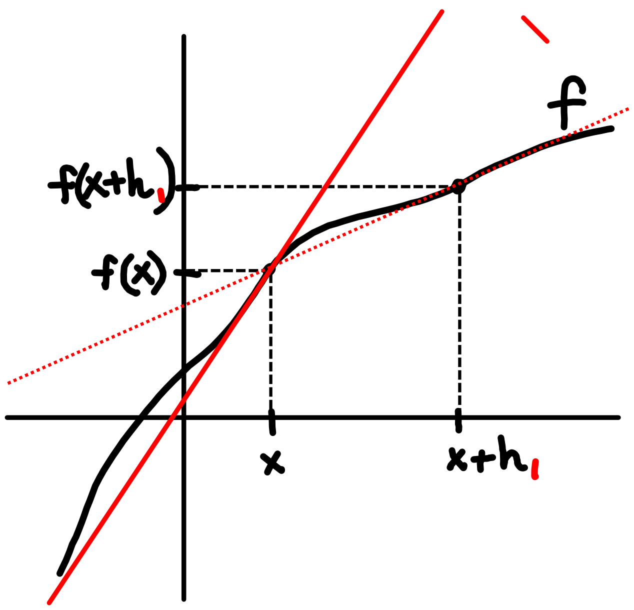
secant line

$$slope = \frac{\Delta y}{\Delta x}$$

$$= \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

the difference quotient



tangent line

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

= slope of tangent line

Δx "delta x"
means change in x

$$\frac{f(x+\Delta x)-f(x)}{\underbrace{\Delta x}} = \frac{f(x+h)-f(x)}{h}$$

treated as a single variable

$f(x) = x^2 + 3x + 2$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+3(x+h)+2-(x^2+3x+2)}{h}$$

1.2

$$f(x) = \frac{x-2}{x^2-4}, \quad x \neq 2, -2$$

What happens to *f(x)* as *x* approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	0.2564	0.2506	0.2500	0.25	0.2499	0.2493	0.2439

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

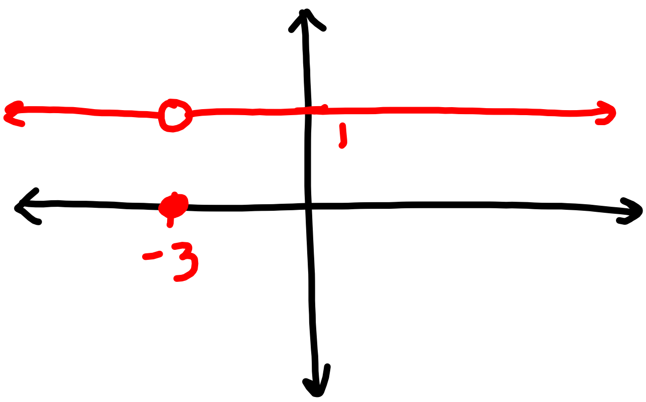
$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

A function can be undefined for a certain value of *c* with the limit as *x* approaches *c* still defined.

$$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$



$$\lim_{x \to -3} f(x) = \boxed{1}$$

$$\lim_{x \to 3} \frac{|x-3|}{x-3} = \left\{ \right.$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$