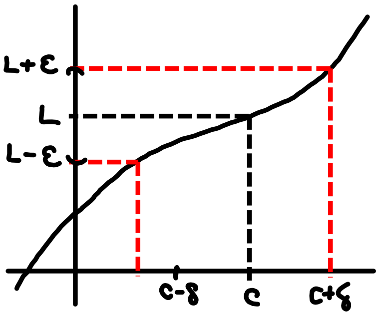
Building up to the $\epsilon - \delta$ Definition of the Limit

ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x \to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L - \varepsilon, L + \varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c - \delta, c)$ or $(c, c + \delta)$.

$0 < |x - c| < \delta$

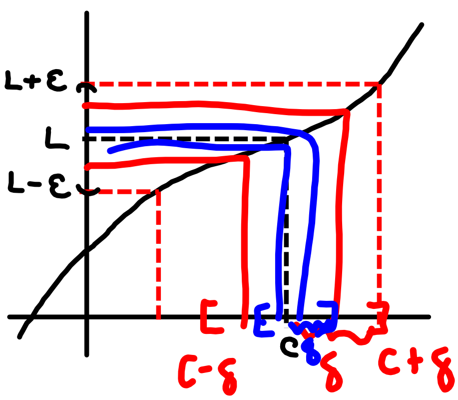
The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.



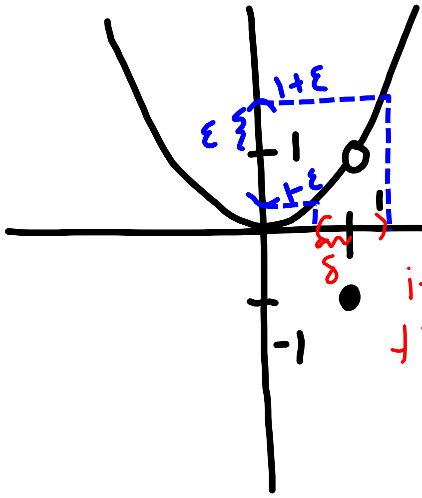
$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

$$\lim_{x\to 1} f(x) = 1$$



if $|x-1| < \delta$,
then
$|f(x) - 1| < \varepsilon$

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$\boxed{f(x) = 2x - 1}$ $\quad \lim_{x\to 4} f(x) = 2(4) - 1 = \boxed{7 = L}$

Find $\lim_{x\to 4} f(x)$ and prove that is the limit using the ε-δ definition. $\boxed{c = 4}$

Let $\varepsilon > 0$ be given. To find $\delta > 0$ so that if $|x-4| < \delta$, then $|f(x) - 7| < \varepsilon$.

$$|f(x) - L| = |2x - 1 - 7| = |2x - 8| = |2(x-4)| = \frac{2|x-4|}{2} < \frac{\varepsilon}{2}$$

$$|x - 4| < \varepsilon/2$$

Take $\delta = \varepsilon/2$.

Then whenever $|x-4| < \delta$, we have that

$$|f(x) - 7| = 2|x-4| < 2\delta = 2 \cdot \varepsilon/2 = \varepsilon, \text{ i.e.}$$

$$|f(x) - 7| < \varepsilon.$$

Hence, $\lim_{x\to 4} f(x) = 7$.

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x+3$; find $\lim_{x \to 1} f(x)$ & find a δ.

$\lim_{x \to 1} f(x) = -5(1)+3 = \boxed{-2 = L}$

Let $\varepsilon > 0$.

$|f(x) - L| = |-5x+3-(-2)| = |-5x+5| = |-5(x-1)|$

$= \frac{5|x-1|}{5} < \frac{\varepsilon}{5} = \delta$

(If $|x-1| < \varepsilon/5$,

$|f(x)-(-2)| = 5|x-1| = 5 \cdot \varepsilon/5 = \varepsilon$)

Prove that the limit is L using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x \to -3} (2x+5)$

$|2x+5-(-1)| = |2x+6|$

$= \frac{2|x-(-3)|}{2} < \frac{\varepsilon}{2} = \delta$

HW #3
due Friday
1.2 p 56
#33, 37, 39, 41