Differential Calculus Quiz #3 09 Nov. 2018

4th Period

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

Find the limit L, and find the largest possible δ for a given ε to prove that L is indeed the limit.

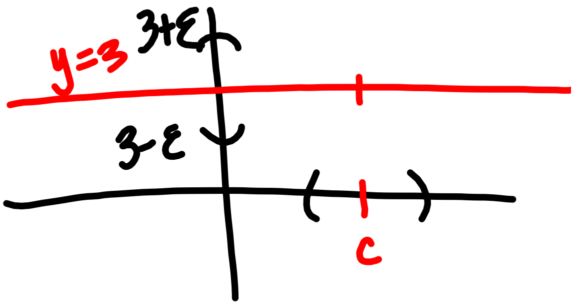
$$\lim_{x \to 4}\left(4 - \frac{x}{2}\right) = 4 - \frac{4}{2} = 4 - 2 = \boxed{2 = L}$$

$$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| = \left|-\frac{1}{2}x + 2\right| =$$

$$= \left|-\frac{1}{2}(x-4)\right| = 2\,\frac{1}{2}|x-4| < \varepsilon \cdot 2$$

$$\boxed{\delta = 2\varepsilon}$$

$$f(x) = 3$$



$$\lim_{x \to c}(3) = 3$$

$$|f(x) - L| = |3 - 3| = 0 < \varepsilon$$

$$\boxed{\delta = \varepsilon}$$

1.3 Evaluating Limits Analytically

If $\lim_{x \to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*, $\lim_{x \to c} f(x) = L$, $\lim_{x \to c} g(x) = K$

1. Constant $\lim_{x \to c} b = b$
2. Identity $\lim_{x \to c} x = c$
3. Polynomial $\lim_{x \to c} x^n = c^n$
4. Scalar Multiple $\lim_{x \to c} [bf(x)] = bL$
5. Sum or Difference $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x \to c} [f(x)g(x)] = LK$
7. Quotient $\lim_{x \to c} \left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x \to c} [f(x)]^n = L^n$

$\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$

$\lim_{x \to c} [f(x) g(x)] = \left[\lim_{x \to c} f(x)\right] \cdot \left[\lim_{x \to c} g(x)\right]$

$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, $\lim_{x \to c} g(x) \neq 0$

$\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

$$\lim_{x\to c} a = a$$

$$\lim_{x\to c} x = c$$

$$\lim_{x\to c} x^n = c^n$$

$$\lim_{x\to 5} (-3) = \boxed{-3}$$

$$\lim_{x\to -\pi} x = \boxed{-\pi}$$

$$\lim_{x\to -1} x^5 = (-1)^5 = \boxed{-1}$$

1.3

12. $\lim_{x\to 1} (3x^3 - 2x^2 + 4) = 3(1)^3 - 2(1)^2 + 4$

$$= \boxed{5}$$

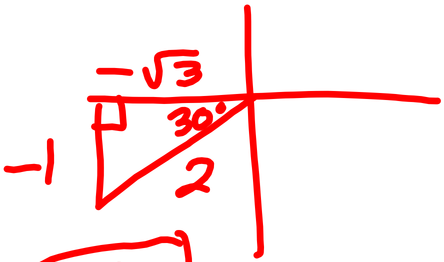
18. $\lim_{x\to 3} \frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = \boxed{-2}$

$x^2 = 4$

$x = \pm 2$

$\sqrt{4} = 2$

30. $\lim_{x \to 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = \boxed{1}$



36. $\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right) = \sec \frac{7\pi}{6} = \boxed{-\frac{2}{\sqrt{3}}}$

38. $\lim_{x \to c} f(x) = \frac{3}{2}$; $\lim_{x \to c} g(x) = \frac{1}{2}$

(a) $\lim_{x \to c} [4f(x)] = 4 \cdot \frac{3}{2} = \boxed{6}$

(b) $\lim_{x \to c} [f(x) + g(x)] = \frac{3}{2} + \frac{1}{2} = \boxed{2}$

(c) $\lim_{x \to c} [f(x)g(x)] = \frac{3}{2} \cdot \frac{1}{2} = \boxed{3/4}$

(d) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2} \cdot \frac{2}{1} = \boxed{3}$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}}$$

$$= \lim_{x\to 3}(x+1) = 3+1 = \boxed{4}$$

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \lim_{x\to 4}(\sqrt{x}+2) = \sqrt{4}+2 = \boxed{4}$$

Given $f(x) = 2x^2 + 3x + 1$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2+3x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h \cancel{+1} \cancel{-2x^2} \cancel{-3x} \cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 + 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$= \lim_{h\to 0} (4x+2h+3) = 4x + 2(0) + 3 = \boxed{4x+3}$$