83. $f(x) = 3x-2$

$$\lim_{h\to 0} \frac{3(x+h)-2-(3x-2)}{h}$$

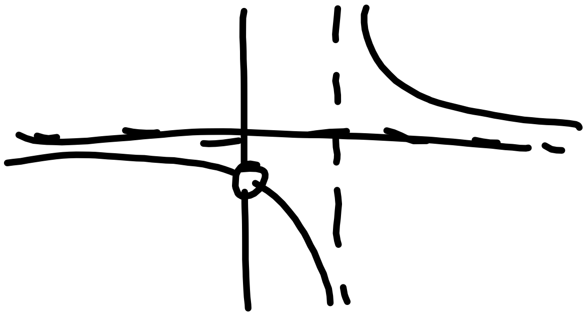
87. $f(x) = \frac{1}{x+3}$

$$\lim_{h\to 0} \frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}$$

Differential Calculus Quiz #4 12 Nov. 2018

4th Period

$$\lim_{x\to 1} \frac{x}{x(x-1)} = \lim_{x\to 1} \frac{1}{x-1}$$



DNE

$$\lim_{x\to 0} \frac{\frac{1}{(x+4)}\cdot\frac{4}{4} - \frac{1}{4}\cdot\frac{x+4}{(x+4)}}{\frac{x}{1}} = \lim_{x\to 0} \frac{4-(x+4)}{4(x+4)}\cdot\frac{1}{x}$$

$$= \lim_{x\to 0} \frac{-\cancel{x}^{1}}{4\cancel{x}(x+4)} = \frac{-1}{4(0+4)} = \frac{-1}{16}$$

$$\lim_{x\to 2} \frac{x^5-32}{x-2}$$

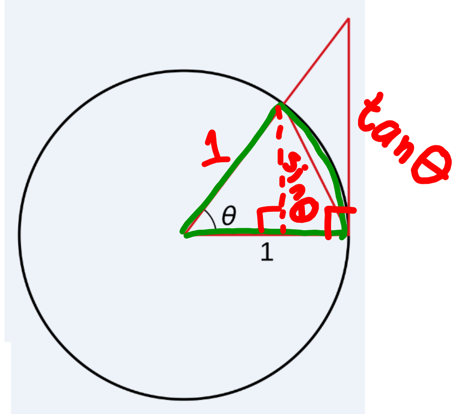
2	1	0	0	0	0	-32
		2	4	8	16	32
	1	2	4	8	16	0

$1x^4+2x^3+4x^2+8x+16$

$$2^4+2(2)^3+4(2)^2+8(2)+16 = 5(16) = \boxed{80}$$

1.3 The Squeeze Theorem

$$\lim_{x\to 0}\frac{\sin x}{x} = ?$$



Area of whole circle $= \pi r^2|_{r=1} = \pi$

$$\frac{\text{Area of whole circle}}{\text{Total angle of circle}} = \frac{\text{Area of sector}}{\theta}$$

$$\frac{\pi}{2\pi} = \frac{\text{Area of sector}}{\theta} \rightarrow \text{Area of sector} = \frac{\theta}{2}$$

Area of outer triangle $\geq$ Area of sector $\geq$ Area of inner triangle

$$\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$$

Multiply through by $\frac{2}{\sin\theta}$

$$\frac{\sin\theta}{2\cos\theta}\cdot\frac{2}{\sin\theta} \geq \frac{\theta}{2}\cdot\frac{2}{\sin\theta} \geq \frac{\sin\theta}{2}\cdot\frac{2}{\sin\theta}$$

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

$$\frac{bd}{ac}\cdot\frac{a}{b} < \frac{c}{d}\cdot\frac{bd}{ac}$$

$$\frac{d}{c} < \frac{b}{a}$$

$$\frac{b}{a} > \frac{d}{c}$$

Take reciprocals and reverse inequalities

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$$

Take limits

$$\lim_{\theta\to 0}\cos\theta \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1$$

$$1 \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq 1$$

$$\Rightarrow \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \boxed{1}$$

$$\lim_{x\to 0}\frac{\sin x}{x} = \boxed{1}$$

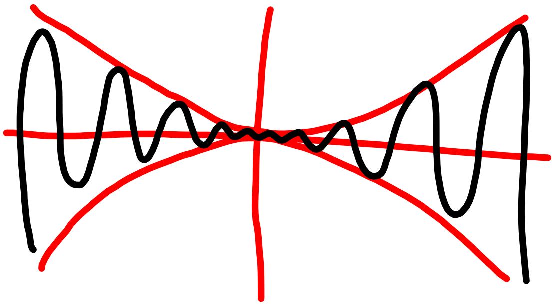
The Squeeze Theorem:

If $f(x) \leq g(x) \leq h(x)$ *and* $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$,

Then $\lim_{x \to c} g(x) = L$.

$-1 \leq \sin x \leq 1$

$-x^2 \leq x^2 \sin x \leq x^2$



Special Limits Derived by Squeeze Theorem:

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1} \; ; \; \boxed{\lim_{x \to 0} \frac{1 - \cos x}{x} = 0}$$

Memorize!!