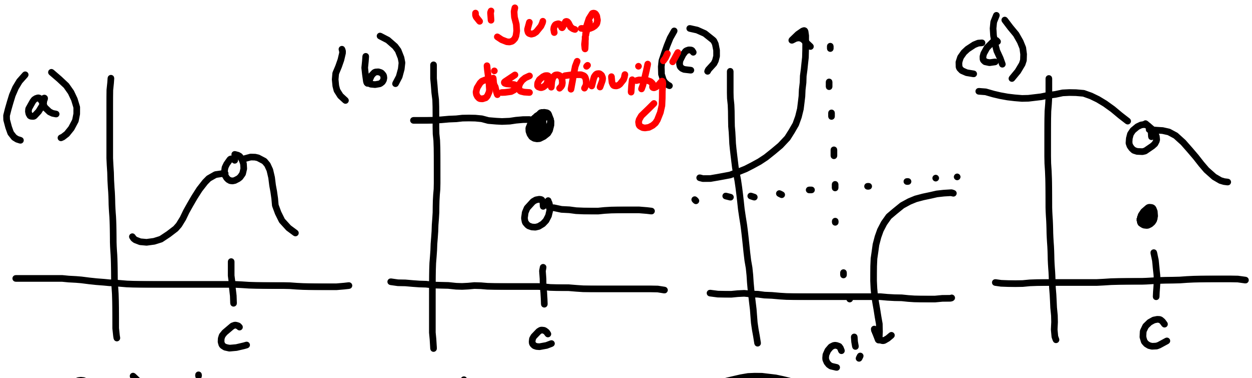
1.4 Continuity and One-Sided Limits



f(c) is undefined

$\lim_{x \to c} f(x)$ does not exist

$\lim_{x \to c} f(x) \neq f(c)$

These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

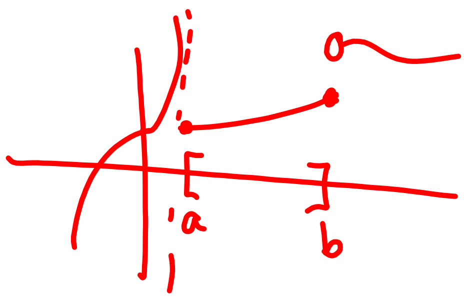
$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is everywhere continuous.

Continuity on a closed interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.



1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$$

removable discontinuity @ $x=2$ (hole)

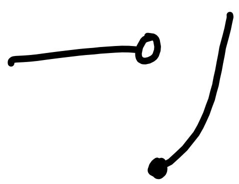
non-removable discontinuity @ $x=-1$ (V.A.)

f is continuous on :

$$\{x \mid x \neq -1, 2\}$$

$$= (-\infty,-1) \cup (-1,2) \cup (2,\infty)$$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$



$$\lim_{x \to 1^-} f(x) = 5$$

$$\lim_{x \to 1} f(x) = \text{DNE}$$

$$\lim_{x \to 1^+} f(x) = 1^2 - 2 = -1$$

f has a non-removable (jump) discontinuity @ $x=1$.

f is continuous on $(-\infty,1) \cup [1,\infty)$

$$f(x) = \frac{|x-2|}{x-2}$$

f has a non-removable (jump) discontinuity @ $x=2$

f is continuous on $(-\infty,2) \cup (2,\infty)$

$$f(x) = \begin{cases} x+6, & x \leq -2 \\ x^2, & -2 < x \leq 3 \\ 8 & x > 3 \end{cases}$$

$-2+6=(-2)^2 \Rightarrow$ f is continuous @ -2

$3^2 \neq 8 \Rightarrow$ f has a non-removable (jump) discontinuity @ $x=3$

f is continuous on $(-\infty, 3] \cup (3,\infty)$

$\lim_{x\to 3^-} f(x) = f(3)$

$\lim_{x\to 3^+} f(x) \neq f(3)$

$$f(x) = \begin{cases} \dfrac{|x-3|}{3-x}, & \boxed{|x-3|>5} \\ x^2-3, & -2 \le x \le 8 \end{cases}$$

$x-3>5$ or $x-3<-5$

$x>8$ or $x<-2$

$$f(x) = \begin{cases} \dfrac{|x-3|}{3-x}, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ \dfrac{|x-3|}{3-x}, & x>8 \end{cases} = \begin{cases} 1, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ -1, & x>8 \end{cases}$$

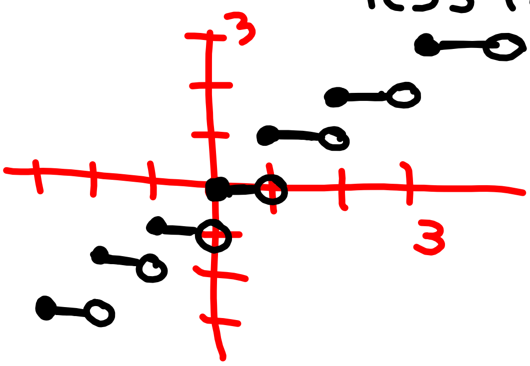
$$\frac{|x-3|}{3-x} = \begin{cases} \dfrac{-(x-3)}{3-x} = 1, & x-3<0,\ x<3 \\ \dfrac{x-3}{3-x} = -1, & x-3>0,\ x>3 \end{cases}$$

f has a non-removable (jump) discontinuity @ $x=8$

f is continuous on $(-\infty,8]\cup(8,\infty)$

The Greatest Integer Function

$[\![x]\!]$ = the greatest integer less than or equal to x



"step function"

22. $\lim_{x \to 2^+} 2x - [\![x]\!]$

$= \lim_{x \to 2^+} 2x - \lim_{x \to 2^+} [\![x]\!]$

$= 2(2) - 2$

$= \boxed{2}$

24. $\lim_{x \to 1} \left(1 - [\![-\frac{x}{2}]\!]\right)$

$= \lim_{x \to 1} (1) - \lim_{x \to 1} [\![-\frac{x}{2}]\!]$

$= 1 - (-1)$

$= \boxed{2}$

$-\frac{1.1}{2}$ $-\frac{0.9}{2}$

$\sim -\frac{1}{2}$

20. $\lim_{x \to \frac{\pi}{2}} \sec x$ DNE