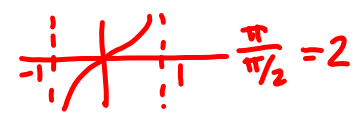


$$\lim_{x \rightarrow 0} \frac{2 \sin 2x}{3x} \cdot \frac{2}{2} = \lim_{x \rightarrow 0} \frac{4}{3} \cdot \frac{\sin 2x}{2x} = \frac{4}{3}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2 \sin 2x}{3x} &= \lim_{x \rightarrow 0} \frac{2(2 \sin x \cos x)}{3x} = \\ &= \lim_{x \rightarrow 0} \frac{4}{3} \cdot \frac{\sin x}{x} \cdot \frac{\cos x}{1} = \frac{4}{3} \end{aligned}$$

52. $f(x) = \tan \frac{\pi x}{2}$ 

discuss the (dis)continuity

f has non-removable vertical asymptote discontinuities at all $x = 2k+1$, $k \in \mathbb{Z}$

f is continuous on all intervals of the form $(2k-1, 2k+1)$

b2. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x - 1$
(both continuous on their domains)
Discuss the continuity of $f(g(x))$.

$f(g(x)) = \frac{1}{\sqrt{x-1}}$ is also continuous on its domain
 $(1, \infty)$

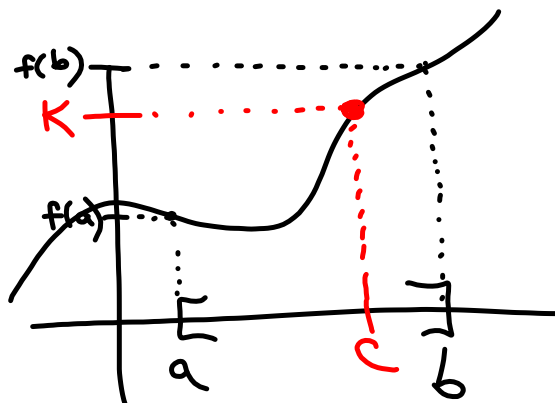
b4. $f(x) = \sin x$; $g(x) = x^2$
discuss the continuity of $f(g(x))$

$$f(g(x)) = \sin(x^2)$$

composition of two functions continuous
on $(-\infty, \infty)$ is cts. on $(-\infty, \infty)$

Intermediate Value Theorem

If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$.



Does the IVT guarantee a zero in the given interval?

$$76. f(x) = x^3 + 3x - 2, [0, 1]$$

$$\left. \begin{array}{l} f(0) = -2 < 0 \\ f(1) = 2 > 0 \end{array} \right\} \Rightarrow \text{IVT applies \& guarantees a zero in } (0, 1)$$

$$84. f(x) = x^2 - 6x + 8; [0, 3]: f(c) = 0$$

$$86. f(x) = \frac{x^2 + x}{x - 1}, \left[\frac{5}{2}, 4 \right], f(c) = 6$$

$$f\left(\frac{5}{2}\right) = \frac{\left(\frac{5}{2}\right)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \frac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \frac{\frac{35}{4}}{\frac{3}{2}} = \frac{35}{4} \cdot \frac{2}{3} = \frac{35}{6} < 6$$

$$f(4) = \frac{4^2 + 4}{4 - 1} = \frac{20}{3} > 6 \Rightarrow \text{IVT guarantees a } c \in \left(\frac{5}{2}, 4\right) \text{ s.t. } f(c) = 6.$$

1.5

Infinite Limits

$$\lim_{x \rightarrow c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$$f(x) = \frac{1}{x(x+3)} \text{ has vertical asymptotes at } x = 0 \text{ and } x = 3$$

$$f(x) = \frac{(x+2)(x+3)}{x(x+3)} \text{ has a vertical asymptote at } x = 0 \text{ and a hole at } x = -3$$

Rules involving infinite limits

$$\text{Let } \lim_{x \rightarrow c} f(x) = \infty \text{ and } \lim_{x \rightarrow c} g(x) = L$$

$$1. \lim_{x \rightarrow c} [f(x) \pm g(x)] = \infty$$

$$2. \lim_{x \rightarrow c} [f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$$

$$3. \lim_{x \rightarrow c} \frac{g(x)}{f(x)} = 0$$

Find the vertical asymptotes (if any).

$$14. f(x) = \frac{-4x}{x^2 + 1} \quad \text{none}$$

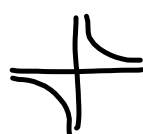
$$24. h(x) = \frac{x^2 - 4}{x^2 + 2x + x + 2} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x^2+1)} \quad \text{none}$$

$x^2(x+2) + (x+2)$

$$28. g(\theta) = \frac{\tan \theta}{\theta}$$

$$42. \lim_{x \rightarrow 0^-} (x^2 - \frac{1}{x}) = \lim_{x \rightarrow 0^-} x^2 - \lim_{x \rightarrow 0^-} \frac{1}{x}$$

$= 0 - (-\infty) = \boxed{\infty}$



$$46. \lim_{x \rightarrow 0} \frac{x+2}{\cot x}$$

Differential Calculus
4th Period

Quiz #6

Name
27 Nov. 2018

1. Discuss the continuity of the function (identify all discontinuities, if any, as removable or non-removable).

$$f(x) = \frac{x^2 - 7x + 10}{x^2 - 3x + 2} = \frac{(x-5)(x-2)}{(x-2)(x-1)}$$

f has a removable discontinuity @ $x=2$ & a non-removable discontinuity @ $x=1$

f is continuous on $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

2 pts
0.25 ea

2. Determine if the Intermediate Value Theorem guarantees a c in the interval $[-2, 3]$ such that $f(c) = -4$, and if so, find all such values of c .

$$f(x) = x^2 - 7x + 2$$

f is continuous on $[-2, 3]$

$f(-2) = (-2)^2 - 7(-2) + 2 = 20$

$f(3) = 3^2 - 7(3) + 2 = -10$

Since $f(3) < -4 < f(-2)$

• IVT guarantees a $c \in (-2, 3)$ s.t. $f(c) = -4$

0.3 ea

$$x^2 - 7x + 2 = -4$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

$x=6, x=1$

$\notin (-2, 3)$

$C=1$

0.25 ea

Find the vertical asymptotes (if any).

28. $g(\theta) = \frac{\tan \theta}{\theta}$

$g(\theta)$ has vertical asymptotes @ $x = \frac{(2k+1)\pi}{2}, k \in \mathbb{Z}$

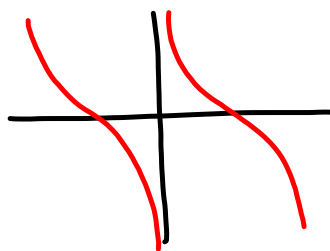
$$\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \cdot \frac{1}{\cos \theta} = 1 \cdot 1 = 1$$

(hole in graph @ $(0, 1)$)

$$46. \lim_{x \rightarrow 0} \frac{x+2}{\cot x} \rightarrow 2 \rightarrow 0$$

$$= \lim_{x \rightarrow 0} (x+2) \tan x$$

$$= 2 \cdot 0 = \boxed{0}$$

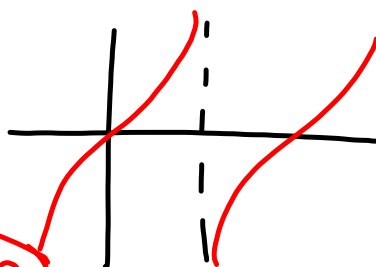


$$48. \lim_{x \rightarrow \frac{1}{2}} x^2 \tan \pi x$$

$$= \left(\lim_{x \rightarrow \frac{1}{2}} x^2 \right) \left(\lim_{x \rightarrow \frac{1}{2}} \tan \pi x \right)$$

$$= \frac{1}{4} \cdot \text{DNE} = \text{DNE}$$

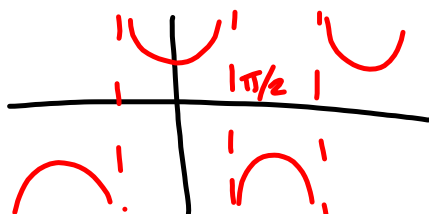
$$\lim_{x \rightarrow \frac{1}{2}^-} x^2 \tan \pi x = \infty, \lim_{x \rightarrow \frac{1}{2}^+} x^2 \tan \pi x = -\infty$$



$$52. \lim_{x \rightarrow 3^+} \sec \frac{\pi x}{6}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^+} \sec x$$

$$= \boxed{-\infty}$$



2.1 The Derivative & The Tangent Line Problem

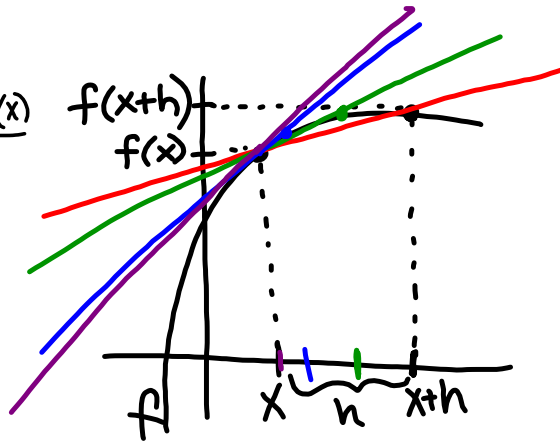
secant line crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h) - f(x)}{x+h - x} = \frac{f(x+h) - f(x)}{h}$$

what happens as $h \rightarrow 0$?

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$



As $h \rightarrow 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it the derivative of f at x .

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x)$ "f prime of x"

$\frac{dy}{dx} = \frac{d}{dx}[y]$ "derivative of y with respect to x"

y' "y prime"

$\frac{d}{dx}[f(x)]$ "the derivative with respect to x of f(x)"

$D_x[y]$ "the partial derivative with respect to x of y"

$$f''(x) = y''$$

$$\frac{d^2 y}{dx^2}$$

$$y^{(n)} = f^{(n)}(x)$$

$$\frac{d^n y}{dx^n}$$

The Derivative

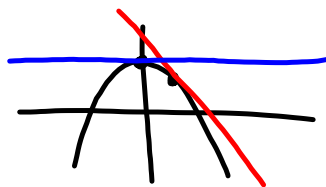
The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = f'(c)$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

8. $g(x) = 5 - x^2$
find slope of tangent line at the points $(2, 1)$ & $(0, 5)$



$$g'(c) = \lim_{h \rightarrow 0} \frac{g(c+h) - g(c)}{h}$$

ⓐ $(2, 1)$ $c=2$; $g(c)=1$

$$g'(2) = \lim_{h \rightarrow 0} \frac{5 - (2+h)^2 - 1}{h} = \lim_{h \rightarrow 0} \frac{5 - (4 + 4h + h^2) - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-4h - h^2}{h} = \lim_{h \rightarrow 0} \frac{h(-4-h)}{h} = \boxed{-4}$$

ⓑ $(0, 5)$:

$$g'(0) = \lim_{h \rightarrow 0} \frac{5 - (0+h)^2 - 5}{h} = \lim_{h \rightarrow 0} \frac{-h^2}{h} = \lim_{h \rightarrow 0} (-h) = \boxed{0}$$

Find the equation of the tangent line to

$f(x) = x^3 - x$ at the point $(2, 6)$.

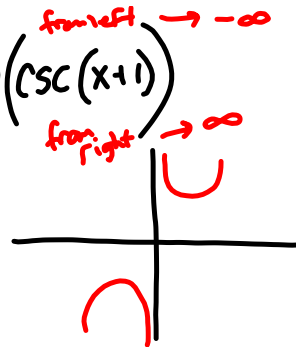
$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - (x+h) - (x^3 - x)}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + \cancel{h^3} - \cancel{x} - h - \cancel{x^3} + \cancel{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2 - 1)}{h} = 3x^2 + 3x(0) + 0^2 - 1 \\
 & \qquad \qquad \qquad f'(x) = 3x^2 - 1
 \end{aligned}$$

slope of tangent
line @ $(2, 6)$: $f'(2) = 3(2)^2 - 1 = \boxed{11}$

$$f(x) = \frac{x}{5 \sin(x+1)} = \left(\frac{1}{5}x\right) (\csc(x+1))$$

$$\lim_{x \rightarrow 1^-} f(x) = \infty$$

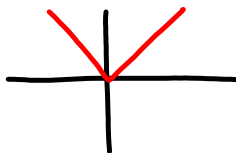
$$\lim_{x \rightarrow 1^+} f(x) = -\infty$$



2.1 Differentiability & Continuity

Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$



All differentiable functions are continuous, but not all continuous functions are differentiable.

e.g. $f(x) = |x|$ Continuous on $(-\infty, \infty)$

$$\lim_{x \rightarrow 0^-} \frac{|x| - |0|}{x - 0} = \lim_{x \rightarrow 0^-} \frac{|x|}{x} = -1$$

$$\lim_{x \rightarrow 0^+} \frac{|x| - |0|}{x - 0} = \lim_{x \rightarrow 0^+} \frac{|x|}{x} = 1$$

} $\lim_{x \rightarrow 0} \frac{|x| - |0|}{x - 0}$ DNE
& derivative defined by that limit does not exist

$$f(x) = \sqrt{x}$$

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

$$f'(0) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x} - \sqrt{0}}{x - 0}$$

$$= \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{x} = \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{x}}$$

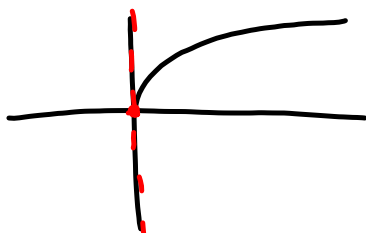
$$= \infty$$

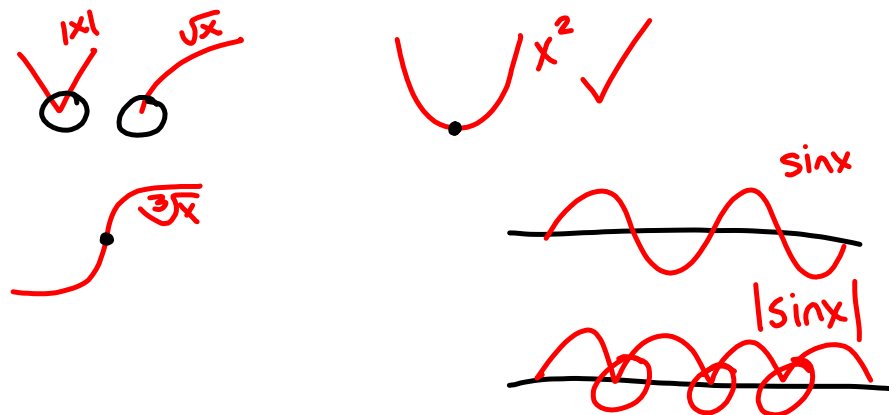
$$\frac{\sqrt{x}}{x} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{x}{x\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$\frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x^1} = \frac{1}{x^{1-1/2}} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$$

$$\frac{x^m}{x^n} = \frac{x^{m-n}}{1} = \frac{1}{x^{n-m}}$$

\sqrt{x} has a vertical tangent line @ $(0,0)$
 $\Rightarrow \sqrt{x}$ is not differentiable @ $(0,0)$





Is f differentiable at $(-3, 0)$? Explain why or why not.

$$f(x) = |x + 3| \quad \text{No.}$$

$$\lim_{x \rightarrow -3^-} \frac{|x+3| - (-3+3)}{x - (-3)} = \lim_{x \rightarrow -3^-} \frac{|x+3|}{x+3} = -1$$

$$\lim_{x \rightarrow -3^+} \frac{|x+3|}{x+3} = 1 \quad f'(-3) = \lim_{x \rightarrow -3} \frac{|x+3|}{x+3} \text{ does not exist}$$

$\Rightarrow f$ is not differentiable @ $(-3, 0)$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

Proof: $f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$

$$\frac{d}{dx}[c] = \lim_{h \rightarrow 0} \frac{c - c}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = \lim_{h \rightarrow 0} (0) = \boxed{0}$$

Examples:

$$f(x) = 3x^2$$

$$f'(x) = 3 \cdot [x^2]' = 3(2x) = \boxed{6x}$$

$$f(x) = \frac{3}{x} = 3x^{-1}$$

$$f'(x) = -3x^{-2} = \boxed{\frac{-3}{x^2}}$$

$$g(x) = 2x^3 - x^2 + 3x$$

$$g'(x) = 6x^2 - 2x + 3$$

$$y = 4x^{3/2} - 5x^4 + 2x^{1/3} - 7$$

$$y' = 4\left(\frac{3}{2}x^{1/2}\right) - 5(4x^3) + 2\left(\frac{1}{3}x^{-2/3}\right) - 0$$

$$= \boxed{6x^{1/2} - 20x^3 + \frac{2}{3}x^{-2/3}}$$

$$= 6\sqrt{x} - 20x^3 + \frac{2}{3\sqrt[3]{x^2}}$$

2. Power Rule for $n \in \mathbb{Q}$, $\frac{d}{dx} [x^n] = nx^{n-1}$

Special case: $\frac{d}{dx} [x] = 1$

Proof:

$$\frac{d}{dx} [x^1] = 1x^0 = 1$$

Recall the binomial expansion:

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \dots + \frac{n!}{k!(n-k)!}a^{n-k}b^k + \dots + b^n$$

$$\begin{aligned} \frac{d}{dx} [x^n] &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots + h^n - \cancel{x^n}}{h} \\ &= \lim_{h \rightarrow 0} \frac{n\cancel{x}^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \dots + h^{n-1}}{1} \\ &= nx^{n-1} + 0 + 0 + \dots + 0 = \boxed{nx^{n-1}} \end{aligned}$$

Examples:

$$\frac{d}{dx} [x^7] = 7x^6 \quad \frac{d}{dx} [x^n] = nx^{n-1}$$

$$\frac{d}{dx} [\pi^3] = 0 \quad (\pi^3 \text{ is a constant and } [c]' = 0)$$

$$\frac{d}{dx} [2e] = 0$$

$$\frac{d}{dx} [\sqrt{x}] = \frac{d}{dx} [x^{1/2}] = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\frac{d}{dx} \left[\frac{1}{x^3}\right] = \frac{d}{dx} [x^{-3}] = -3x^{-4} = \boxed{-\frac{3}{x^4}}$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{12(1-\cos x)}{x^2} \cdot \frac{1+\cos x}{1+\cos x} \\
 &= \lim_{x \rightarrow 0} \frac{12(1-\cos^2 x)}{x^2(1+\cos x)} = \lim_{x \rightarrow 0} \frac{12}{1+\cos x} \cdot \frac{\overset{\uparrow 1}{\sin^2 x}}{x^2} \\
 &= \frac{12}{1+1} = 6
 \end{aligned}$$

$$f(x) = \begin{cases} -4 \frac{\sin x}{x}, & x < 0 \\ a + 7x, & x \geq 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} \left(-4 \frac{\sin x}{x} \right) = \lim_{x \rightarrow 0^+} (a + 7x)$$

$$\boxed{-4 = a}$$

$$\lim_{x \rightarrow -5^-} 6 = \lim_{x \rightarrow -5^+} (ax+b) \rightarrow 6 = -5a+b$$

$$\lim_{x \rightarrow 1^-} (ax+b) = \lim_{x \rightarrow 1^+} (-6) \rightarrow a+b = -6$$

$$\begin{array}{l} b = -6 - (-2) \quad 5a - b = -6 \\ = -4 \quad \hline 6a = -12 \\ a = -2 \end{array}$$

$$\frac{x^2 - 5x}{x-3} = 6$$

$$(x=9), 2$$

$$x^2 - 5x = 6(x-3)$$

$$x^2 - 5x = 6x - 18$$

$$x^2 - 11x + 18 = 0$$

$$(x-9)(x-2) = 0$$

$$\lim_{x \rightarrow c^-} (4 - x^2) = \lim_{x \rightarrow c^+} x$$

$$4 - c^2 = c$$

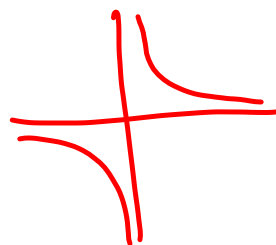
$$0 = c^2 + c - 4$$

$$c = \frac{-1 \pm \sqrt{1^2 - 4(1)(-4)}}{2(1)} = \frac{-1 \pm \sqrt{17}}{2}$$

$$\lim_{x \rightarrow 0^-} x^2 - \lim_{x \rightarrow 0^-} \frac{1}{x}$$

$$0 - (-\infty)$$

$$= \boxed{\infty}$$



$$g(x) = 9 - x^2$$
$$g'(x) = -2x$$
$$m = g'(4) = \boxed{-8}$$

$$\lim_{h \rightarrow 0} \frac{\frac{2}{x+h-3} - \frac{2}{x-3}}{h} = \lim_{h \rightarrow 0} \frac{2(x-3) - 2(x+h-3)}{h(x-3)(x+h-3)}$$
$$= \lim_{h \rightarrow 0} \frac{\cancel{2x} - \cancel{6} - \cancel{2x} - \cancel{2h} + \cancel{6}}{h(x-3)(x+h-3)} = \frac{-2}{(x-3)(x-3)}$$

$$f(x) = x^2 + 5x + 2, \quad (-5, 2)$$

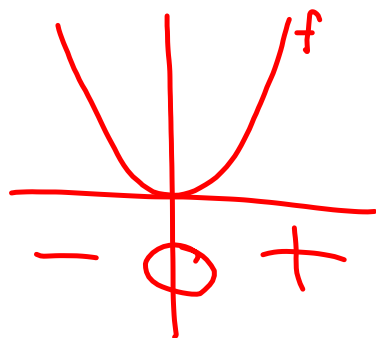
$$f'(x) = 2x + 5$$

$$m = f'(-5) = 2(-5) + 5 = -5$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -5(x - (-5))$$

$$y = -5x - 23$$



$$y = x^2$$
$$y' = 2x$$

$$\begin{aligned}
 f(x) &= x^2 - 9 ; x = 5 \\
 \lim_{x \rightarrow 5} \frac{(x^2 - 9) - (5^2 - 9)}{x - 5} &= \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} \\
 &= \lim_{x \rightarrow 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}} = 5 + 5 = \boxed{10}
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 2} \frac{\frac{3}{x^2} - \frac{3}{2^2}}{x - 2} &= \lim_{x \rightarrow 2} \frac{3 \cdot 4 - 3 \cdot x^2}{(x - 2) \cdot 4x^2} \\
 &= \lim_{x \rightarrow 2} \frac{-3(x^2 - 4)}{(x - 2) \cdot 4x^2} = \frac{-3(x-2)(x+2)}{\cancel{(x-2)} \cdot 4x^2} \\
 &= \frac{-3(2+2)}{4(2^2)} = -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \frac{2x}{x+4} \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{2(x+h)}{(x+h)+4} - \frac{2x}{x+4}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{2(x+h) \cdot \frac{x+4}{x+4} - \frac{2x \cdot \frac{x+h+4}{x+h+4}}{h}}{h}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x+h)(x+4) - 2x(x+h+4)}{(x+4)(x+h+4)} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x^2 + 4x + hx + 4h) - 2x^2 - 2xh - 8x}{h(x+4)(x+h+4)} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{8x} + \cancel{2hx} + \cancel{8h} - \cancel{2x^2} - \cancel{2xh} - \cancel{8x}}{h(x+4)(x+h+4)} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{8h}}{\cancel{h}(x+4)(x+h+4)} = \boxed{\frac{8}{(x+4)^2}}
 \end{aligned}$$

$$f(x) = \begin{cases} x^2 - 5 & , x \leq c \\ \frac{\sqrt{x} - 5}{x - 5} & , x > c \end{cases}$$

$$\begin{aligned}
 \lim_{x \rightarrow c^-} (x^2 - 5) &= \lim_{x \rightarrow c^+} \frac{\sqrt{x} - 5}{x - 5} \cdot \frac{\sqrt{x} + 5}{\sqrt{x} + 5} \\
 c^2 - 5 &= \lim_{x \rightarrow c^+} \frac{x - 25}{(x-5)(\sqrt{x} + 5)}
 \end{aligned}$$

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,
for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \in \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $c \in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

$$1. \frac{d}{dx}[\sin x] = \cos x$$

$$2. \frac{d}{dx}[\cos x] = -\sin x$$

$$3. \frac{d}{dx}[\tan x] = \sec^2 x$$

$$4. \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$5. \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$6. \frac{d}{dx}[\csc x] = -\csc x \cot x$$