$f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

$3x(x-1) = 0$

$x = 0, 1$

$f(-1) = -5/2 \leftarrow$ abs. min

$f(0) = 0$

$f(1) = -1/2$

$f(2) = 2 \leftarrow$ abs max

$f(x) = x(x^2 - x - 2) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$\frac{f(1) - f(-1)}{1-(-1)} = \frac{1-1-2 + [+1(1+1-2)]}{2} = \frac{-2+0}{2} = -1$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -1/3 \quad x = 1$

$$y = \frac{x^2}{x^2-9}$$

$$y' = \frac{(x^2-9)2x - x^2(2x)}{(x^2-9)^2}$$

$$= \frac{2x(x^2-9-x^2)}{(x^2-9)^2}$$

$$y' = \frac{-18x}{(x^2-9)^2}$$

$$[(x-3)(x+3)]^2$$

$f'(-4)$, $f'(-1)$, $f'(1)$, $f'(4)$

$+ \quad -3 \quad + \quad 0 \quad - \quad 3 \quad -$

$\nearrow \quad \nearrow \quad \searrow \quad \searrow$

f is increasing on $(-\infty,-3)\cup(-3,0)$
decreasing on $(0,3)\cup(3,\infty)$
f has a relative max @ $(0,0)$
& no relative min

$$f(x) = \frac{x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

$f''(-5)$, $f''(-1)$, $f''(1)$, $f''(5)$

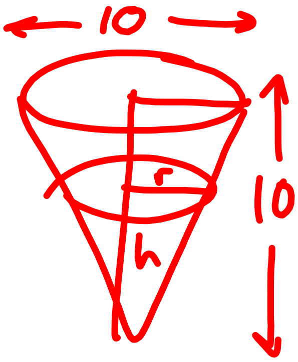
$- \quad -\sqrt{3} \quad + \quad 0 \quad - \quad \sqrt{3} \quad +$

$\frown \quad \smile \quad \frown \quad \smile$

$$f''(x) = \frac{(x^2+1)^2(-2x) - (1-x^2)[2(x^2+1)(2x)]}{(x^2+1)^4}$$

$$= \frac{-2x(x^2+1)[x^2+1+2(1-x^2)]}{(x^2+1)^4} = \frac{-2x(3-x^2)}{(x^2+1)^3}$$

f is concave up on $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$; down on $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$
infl. pts @ $(-\sqrt{3}, \frac{-\sqrt{3}}{4})$, $(0,0)$ & $(\sqrt{3}, \sqrt{3}/4)$



$\frac{dV}{dt} = 5 \text{ ft}^3/\text{min}, \quad \frac{dh}{dt} = ?$ when $h = 3$

$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 \cdot h$

$\frac{r}{h} = \frac{5}{10} = \frac{1}{2}$

$r = \frac{h}{2}$

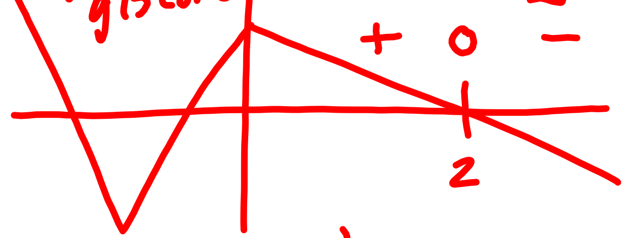
$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{\frac{\pi}{4}(3)^2} = \boxed{\frac{20}{9\pi}}$

g has rel max on 2

g is concave down



$f = g'$

$g'(-1) = -1/2$

$g'(1) = 1/2$

$g''(-1) = 3/2$

$g''(1) = -1/2$

$y - 4 = 1/2(x - 1)$

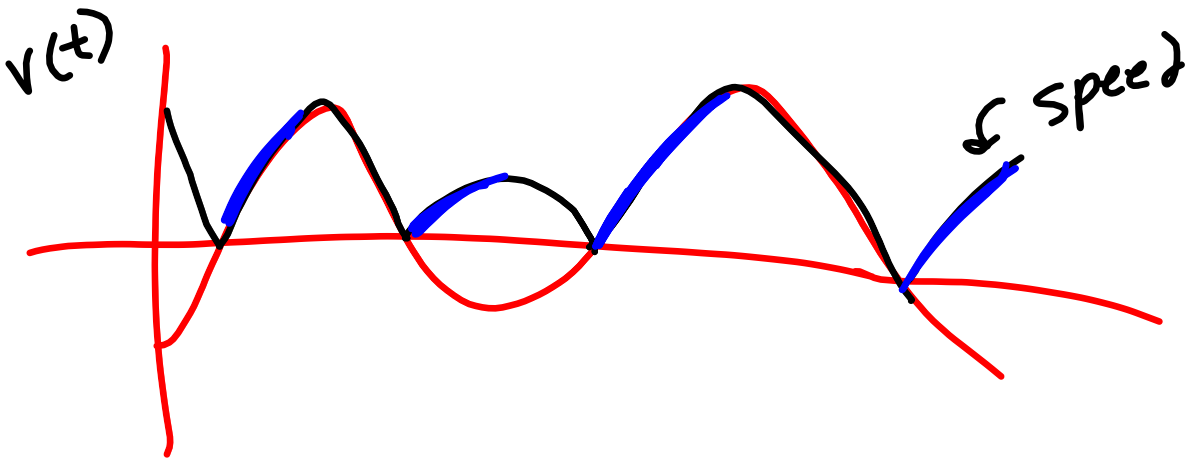
$$g''(x) = \begin{cases} -3/2, & x < -2 \\ 3/2, & -2 \le x \le 0 \\ -1/2, & x > 0 \end{cases}$$

$$a(t) = v'(t) = -\frac{2t-4}{(t-2)^4+1}$$

(b) $(1,2)$

(c) $(1,2) \cup (3,\infty)$

t
$v(t)$
$a(t)$
$s(t)$ + + +



$f'(x) = (x^2 - 4)g(x)$; $g(x) < 0$

$f'(x) = (x-2)(x+2)\underline{g(x)}$

$f'(-3)$ $f'(0)$ $f'(3)$

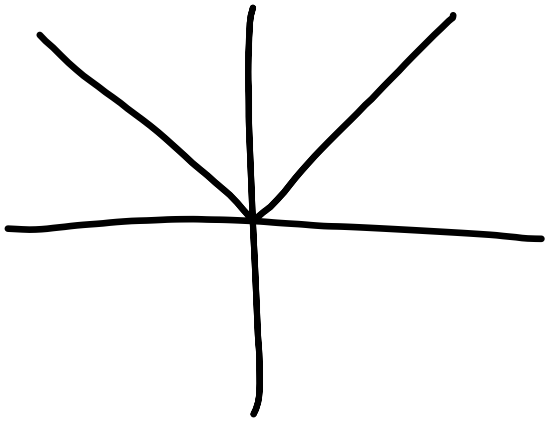
$-$ $\quad -2 \quad$ $+$ $\quad 2 \quad$ $-$

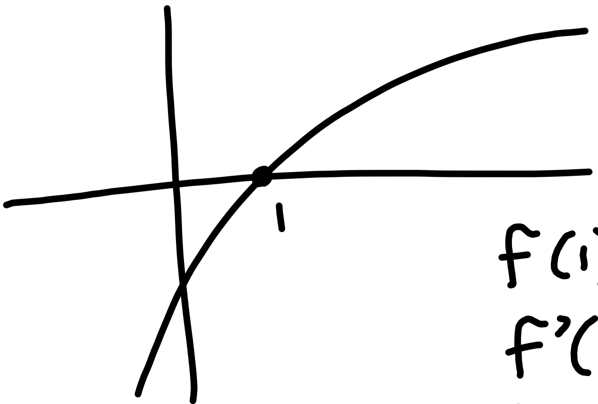
min max

$\frac{dr}{dt} = -0.1$; $C = 2\pi r$; $\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = C \cdot (-0.1)$



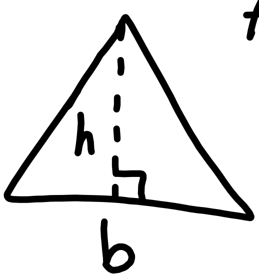


$f''(1) < f(1) < f'(1)$

$f(1) = 0$

$f'(1) > 0$

$f''(1) < 0$



$$A = \frac{1}{2}bh \qquad \frac{db}{dt} = 3 \,;\ \frac{dh}{dt} = -3$$

$$\frac{dA}{dt} = \frac{1}{2}b \cdot \frac{dh}{dt} + \frac{1}{2}h \cdot \frac{db}{dt}$$

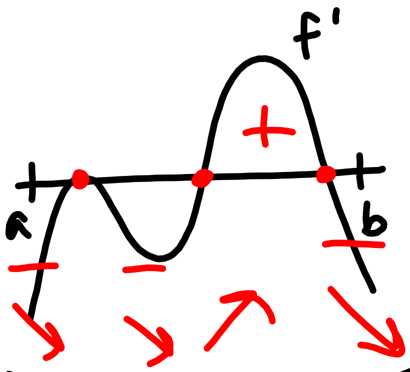
$$\frac{dA}{dt} = -\frac{3}{2}b + \frac{3}{2}h$$

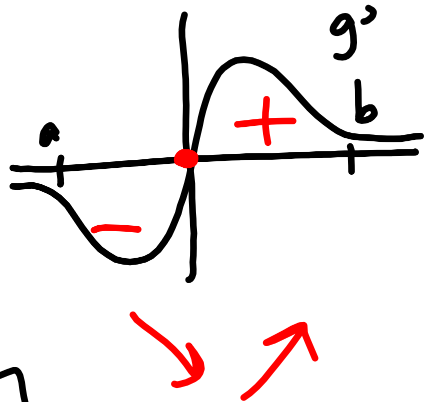
$$= \frac{3}{2}(-b+h)$$

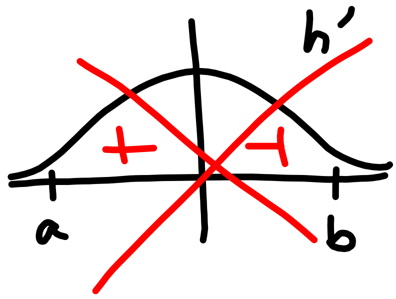
A incr when

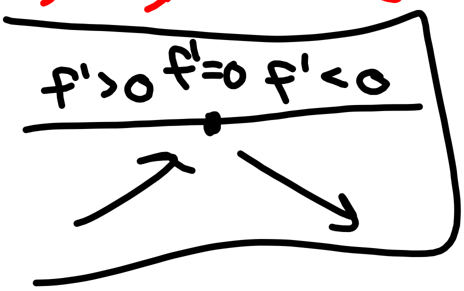
$\frac{dA}{dt} > 0 \Rightarrow \frac{3}{2}(-b+h) > 0 \Rightarrow -b+h > 0 \Rightarrow h > b$

decr. when $\frac{dA}{dt} < 0$









$f(x) = \tan^2 x$; $(\pi/4, 1)$ (x_1, y_1)

$f'(x) = 2\tan x \cdot \sec^2 x$

$m = f'(\frac{\pi}{4}) = 2\tan\frac{\pi}{4} \cdot \sec^2\frac{\pi}{4}$

$= 2 \cdot 1 \cdot 2 = 4$

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x - \pi/4)$

$\boxed{y = 4x - \pi + 1}$

$f(x) = \frac{1}{2}x^3 - 5x$; $(2, -6)$

$f'(x) = \frac{3}{2}x^2 - 5$

$m = f'(2) = \frac{3}{2}(4) - 5 = 1$

$y - (-6) = 1(x - 2)$

$y + 6 = x - 2$

$y = x - 2 - 6$

$\boxed{y = x - 8}$

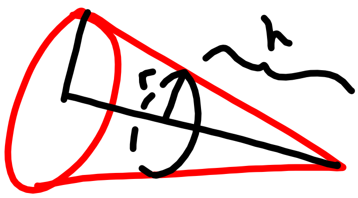
$$2xy = 9$$

$$xy = \frac{9}{2}$$

$$1 \cdot y + x \cdot y' = 0$$

$$xy' = -y$$

$$y' = \frac{-y}{x}$$



$$V = \frac{\pi}{3} r^2 h \; ; \; r = \frac{h}{2} \; ; \; \frac{dh}{dt} = ?$$

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{3}\left(\frac{h^2}{4}\right) h$$

$$V = \frac{\pi h^2}{12} \cdot \frac{h}{1} = \frac{\pi}{12} \cdot h^3$$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{12}\,4}(\cancel{3}h^2) \cdot \frac{dh}{dt}$$

$$\frac{\frac{dV}{dt}}{\frac{\pi}{4} h^2} = \frac{dh}{dt}$$

$$g(t)=\frac{t^2}{t^2+2}, \ [-3,3]$$

$g(-3)=9/11$

$g(0)=0 \leftarrow$ min

$g(3)=9/11 \leftarrow$ max

$$g'(t)=\frac{(t^2+2)(2t)-t^2(2t)}{(t^2+2)^2}$$

$$=\frac{2t\left[t^2+2-t^2\right]}{(t^2+2)^2}=\frac{4t}{(t^2+2)^2}$$

$\cancel{t^2+2=0}$

$\cancel{t^2=-2}$

$\cancel{t=\pm\sqrt{2}\,i}$

critical #'s: 0

side question about relative extrema

$$\begin{array}{c|c} g'(-1) & g'(1) \\ \hline - & + \\ \searrow & \nearrow \end{array}$$

0

g has a relative minimum @ (0, 0)

$$f(x)=x^3-3x$$

$$f'(x)=3x^2-3$$

$$f''(x)=6x$$

$$\begin{array}{c|c} f''(-1) & f''(1) \\ \hline - & + \\ \frown & \smile \end{array}$$

0

inflection pt @ $(0, f(0))$ $= (0,0)$

$x^2+4y=xy$

$4y-xy=-x^2$

$y(4-x)=-x^2$

$y=\frac{-x^2}{4-x}=\frac{x^2}{x-4}$

$y'=\frac{(x-4)(2x)-x^2(1)}{(x-4)^2}$

$=\frac{x(2x-8-x)}{(x-4)^2}$

$=\frac{x(x-8)}{(x-4)^2}$ $\boxed{x=0,8}$

$2x+4y'=1\cdot y+xy'$

$2x-y=xy'-4y'$

$2x-y=y'(x-4)$

$\frac{2x-y}{x-4}=y'=0$ when $2x-y=0$

$2x=y$

$x^2+4y=xy$

$x^2+4(2x)=x(2x)$

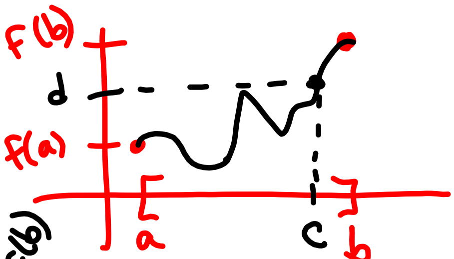
$x^2+8x=2x^2$

$0=x^2-8x$

$=x(x-8)$

$\boxed{x=0,8}$

IVT: If f is continuous on $[a,b]$, then for any real # d such that either $f(a)<d<f(b)$ or $f(a)>d>f(b)$, there exists at least one $c\in(a,d)$ such that $f(c)=d$.



Test f(a) & f(b)

$f(x)=x^2-6x+8$; $[\overset{a}{0},\overset{b}{3}]$; $f(\overset{c}{c})=\overset{d}{0}$

$\left.\begin{array}{l} f(0)=8>0 \leftarrow \text{compare to } d \\ f(3)=9-18+8=-1<0 \end{array}\right\} \Rightarrow$ IVT guarantees $c\in(0,3)$ s.t. $f(c)=0$

set $f(x)=d$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x=2,4$

f is cts.

x	1	2	3	4
$f(x)$	3	7	-1	3

on which intervals does IVT guarantee $f(x)=2$?

$(2,3) \cup (3,4)$