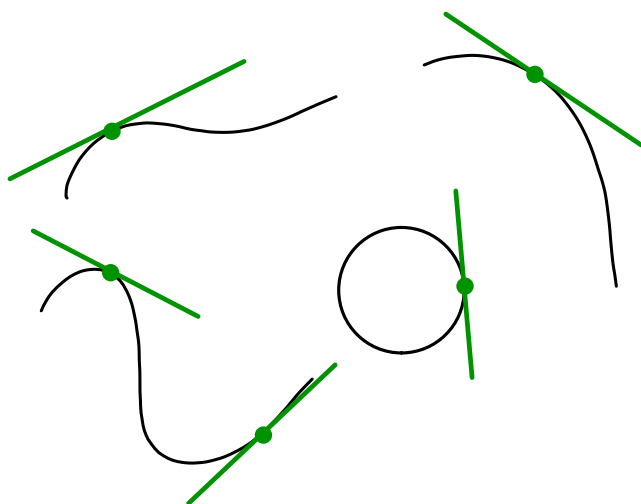
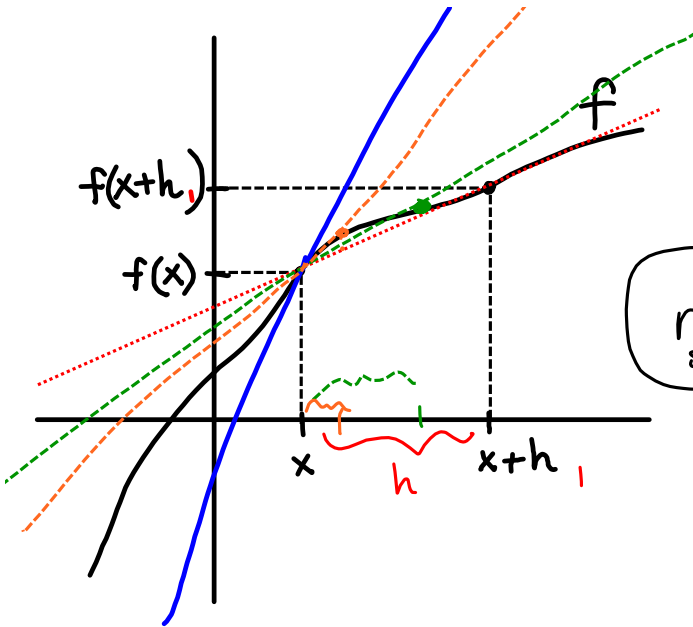


as x approaches..	$f(x)$ approaches...
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines



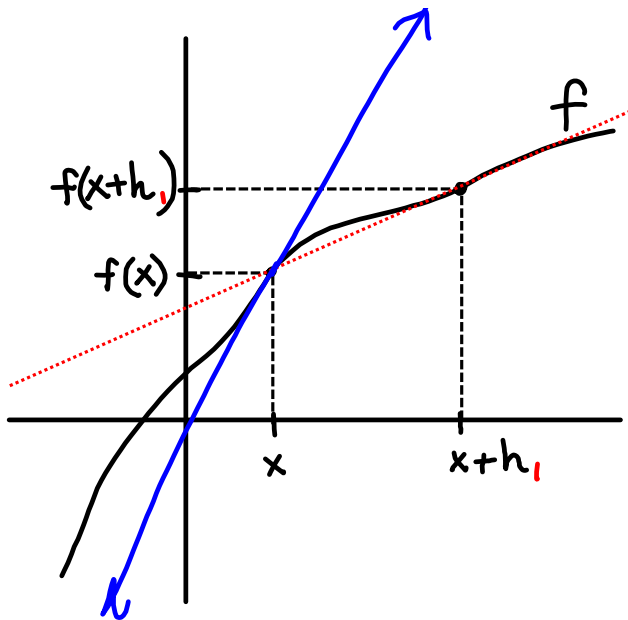


secant line

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h - x}$$

$$m_{\text{secant}} = \frac{f(x+h) - f(x)}{h}$$

difference quotient



tangent line

$$m_{\text{tangent}} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Δx "delta x"
means change in x

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

↑ treated as a single variable

1.2

$$f(x) = \frac{x-2}{x^2-4}, \quad x \neq 2, -2$$

$$= \frac{\cancel{x-2}}{(\cancel{x-2})(x+2)} = \frac{1}{x+2}$$

What happens to $f(x)$ as x approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	0.2564	0.2506	0.2501	0.25	0.2499	0.2494	0.2439

Informal Description of the Limit

watch epsilon-delta
videos on Khan Academy

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c , is L .

$$\lim_{x \rightarrow c} f(x) = L$$

Note: the existence or nonexistence of $f(x)$ at $x=c$ has no bearing on the existence of the limit as x approaches c .

A function can be undefined for a certain value of c with the limit as x approaches c still defined.

$$\lim_{x \rightarrow -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$

$$\lim_{x \rightarrow -3} f(x) = \boxed{1}$$

