Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x\to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

$$\lim_{x\to 3} \frac{|x-3|}{x-3}$$

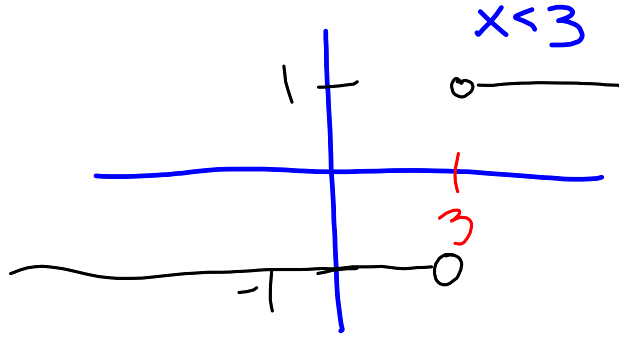
does not exist

$$\frac{|x-3|}{x-3} = \begin{cases} \dfrac{x-3}{x-3} = 1, & x-3>0 \\ & x>3 \\ \dfrac{-(x-3)}{x-3} = -1, & x-3<0 \\ & x<3 \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$$

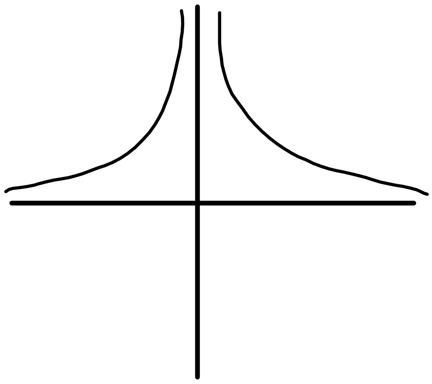
$|2| = 2$

$|-2| = -(-2) = 2$

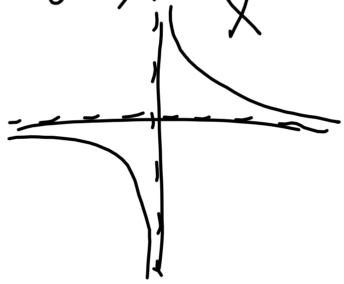


$$\lim_{x\to 3^-} \frac{|x-3|}{x-3} = -1 \; ; \; \lim_{x\to 3^+} \frac{|x-3|}{x-3} = 1$$

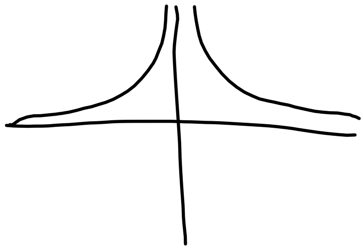
$$\lim_{x\to 0} \frac{1}{x^4} = \boxed{\infty}$$



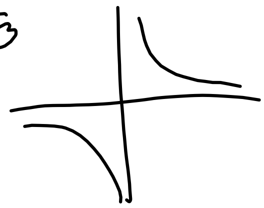
$y = \frac{1}{x} \approx \frac{1}{x^{odd}}$



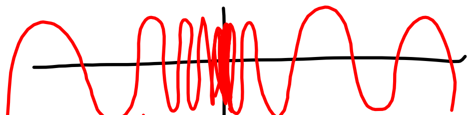
$y = \frac{1}{x^2} \approx \frac{1}{x^{even}}$

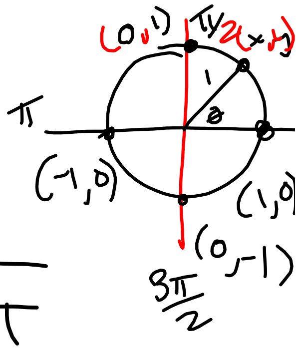


$y = \frac{1}{x^3}$



$$\lim_{x\to 0} \sin\frac{1}{x}$$ does not exist

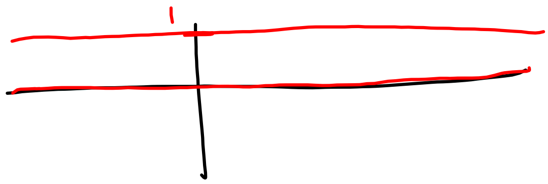




x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin\frac{1}{x}$	$\frac{1}{x} = \frac{\pi}{2}$ $\sin\frac{\pi}{2}$ 1	$\sin\frac{3\pi}{2}$ -1	$\sin\frac{5\pi}{2}$ 1	-1	1	-1

"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$



$\lim_{x \to c} f(x)$ does not exist for any c

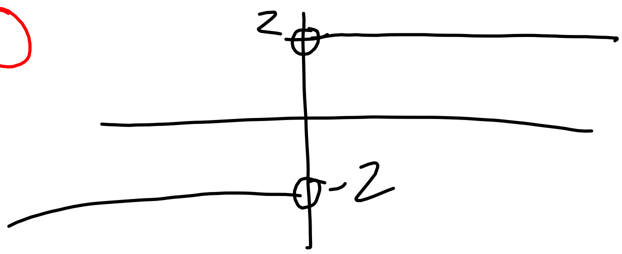
$$\lim_{x \to 0} \frac{|2x|}{x}$$

$$\frac{|2x|}{x} = \begin{cases} \frac{2\cancel{x}}{\cancel{x}} = 2, & 2x > 0,\ x > 0 \\ \frac{-2\cancel{x}}{\cancel{x}} = -2, & 2x < 0,\ x < 0 \end{cases}$$

$$\lim_{x \to 0^+} f(x) = 2$$

$$\lim_{x \to 0^-} f(x) = -2$$

$$\lim_{x \to 0} f(x) = \text{DNE}$$



ε = epsilon

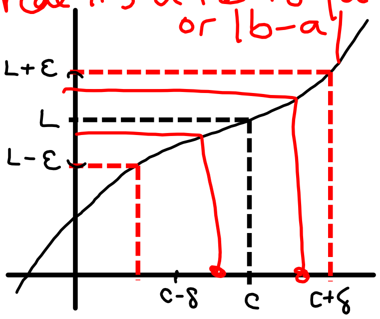
δ = delta

Building up to the $\epsilon - \delta$ Definition of the Limit

Translating the "informal description": $\lim_{x\to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.

distance between real #'s a & b is $|a-b|$ or $|b-a|$



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$0 < |x-c| < \delta$

The first inequality guarantees that $x \neq c$.

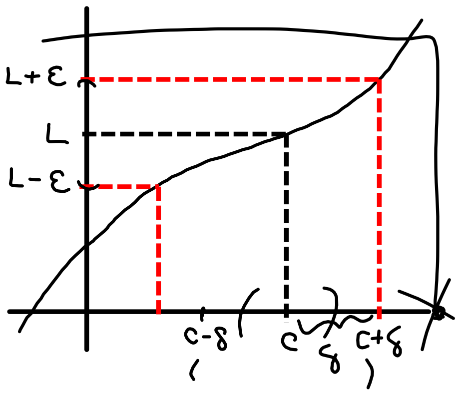
$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x\to c} f(x) = L$$

dependent on ε

means that for each $\varepsilon > 0$ (we choose), there exists a $\delta > 0$ such that if $0 < \underbrace{|x-c|}_{\text{the distance between } x \,\&\, c} < \delta$, then $\underbrace{|f(x)-L|}_{\text{the distance between } f(x) \,\&\, L} < \varepsilon$.

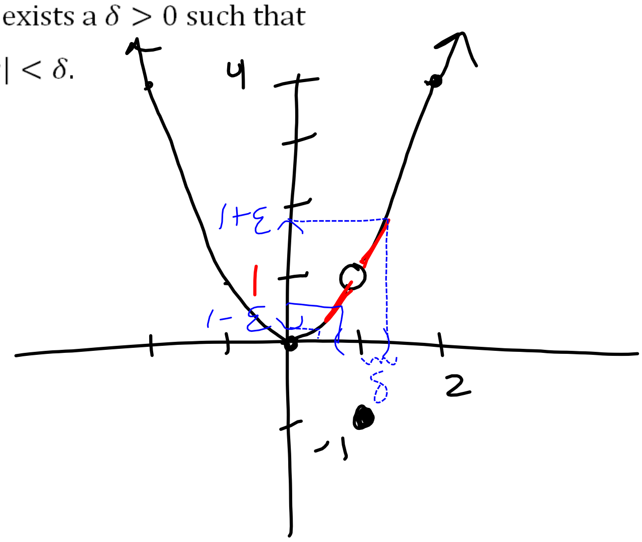


$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = 1$$



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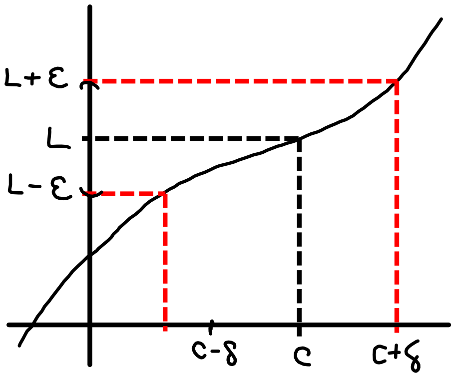

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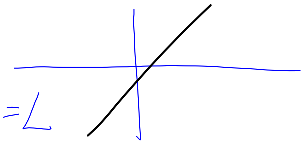

$f(x) = 2x - 1$ $\qquad \lim_{x \to 4} (2x-1) = 7 = L$

Find $\lim_{\substack{x \to 4 \\ = c}} f(x)$ and prove that is the limit using the ε-δ definition.

Let $\varepsilon > 0$ be given.
To find $\delta > 0$ so that whenever $|x - c| = |x - 4| < \delta$ we are guaranteed that
$|f(x) - L| = |2x - 1 - 7| < \varepsilon$.

$|2x - 8| = 2|x - 4| < \varepsilon$

$|x - 4| < \varepsilon/2 \longleftarrow$ take $\delta = \varepsilon/2$.

So then if $|x - 4| < \varepsilon/2$, we get that $|2x - 8| = 2|x - 4| < 2 \cdot \varepsilon/2 = \varepsilon$

Hence $\lim_{x \to 4} (2x - 1)$ is indeed 7.

1.2 # 1-5 odd
15-21 odd
23, 24

due Fri