1.2 # 1-5 odd
15-21 odd
23, 24

due Fri

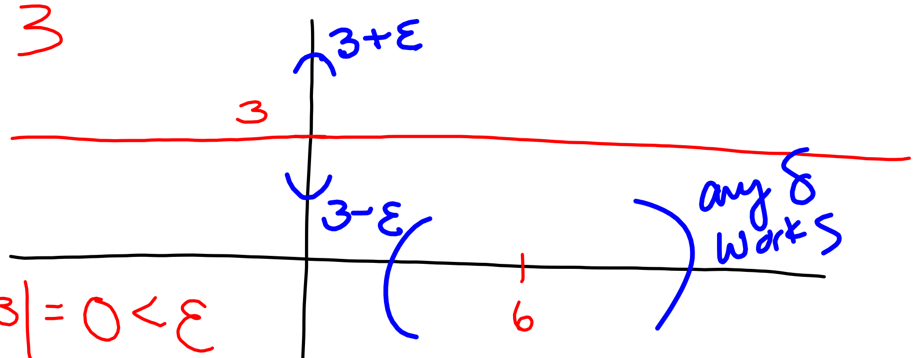
37, 39, 41 $\longrightarrow$ find δ

$f(x) = 3$

$\lim_{x \to 6} f(x) = 3$

Let $\varepsilon > 0$ be given

$|f(x) - L| = |3 - 3| = 0 < \varepsilon$



$\delta = \varepsilon$

1.3 Evaluating Limits Analytically

If $\lim_{x \to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant $\lim_{x\to c} b = b$
2. Identity $\lim_{x\to c} x = c$
3. Polynomial $\lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x\to c}[f(x)g(x)] = LK$
7. Quotient $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x\to c}[f(x)]^n = L^n$

$\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

$\lim_{x\to c}[f(x) g(x)] = \left[\lim_{x\to c} f(x)\right] \cdot \left[\lim_{x\to c} g(x)\right]$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, $\lim_{x\to c} g(x) \neq 0$

$\lim_{x\to c} [f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

$\lim_{x \to c} a = a$

$\lim_{x \to c} x = c$

$\lim_{x \to c} x^n = c^n$

$\lim_{x \to 5} (-3) = -3$

$\lim_{x \to -\pi} x = -\pi$

$\lim_{x \to -1} x^5 = (-1)^5 = -1$

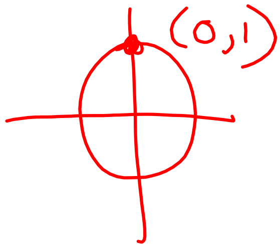
1.3

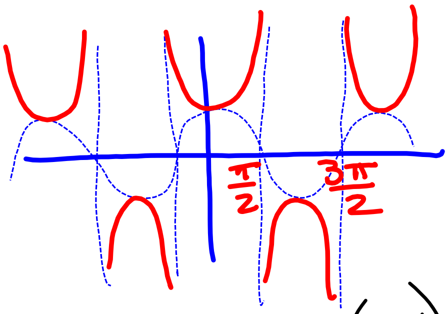
12. $\lim_{x \to 1} (3x^3 - 2x^2 + 4)$

$= 3 \cdot 1 - 2 \cdot 1 + 4 = \boxed{5}$

18. $\lim_{x \to 3} \frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \boxed{-2}$

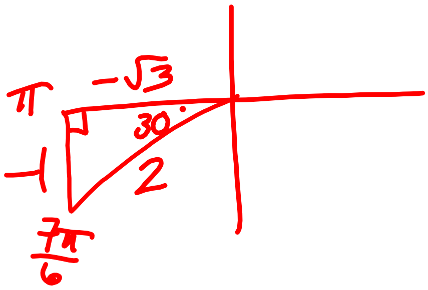
30. $\lim_{x\to 1} \sin\frac{\pi x}{2} = \sin\frac{\pi}{2}$





$= \boxed{1}$

36. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right) = \sec\frac{7\pi}{6}$



$= \boxed{-\frac{2}{\sqrt{3}}}$

38. $\lim_{x\to c} f(x) = \frac{3}{2}$; $\lim_{x\to c} g(x) = \frac{1}{2}$

(a) $\lim_{x\to c}\left[4f(x)\right] = 4\cdot\lim_{x\to c} f(x) = 4\cdot\frac{3}{2} = \boxed{6}$

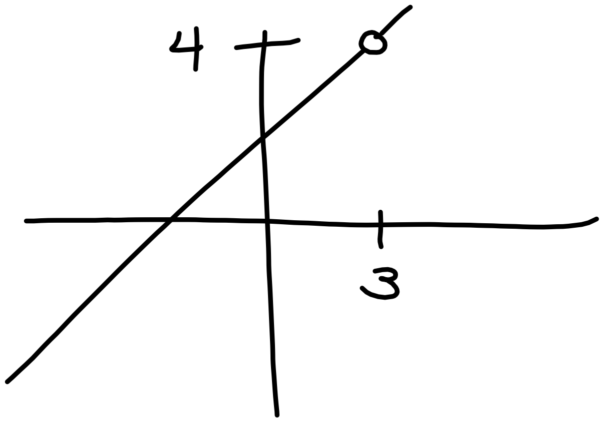
(b) $\lim_{x\to c}\left[f(x)+g(x)\right] = \frac{3}{2}+\frac{1}{2} = \frac{4}{2} = \boxed{2}$

(c) $\lim_{x\to c}\left[f(x)g(x)\right] = \frac{3}{2}\cdot\frac{1}{2} = \boxed{\frac{3}{4}}$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2}\cdot\frac{2}{1} = \boxed{3}$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3} \frac{(\cancel{x-3})(x+1)}{\cancel{x-3}}$$

$$= \lim_{x\to 3} (x+1) = 3+1 = \boxed{4}$$



$$\lim_{x\to 4} \frac{(x-4)}{\sqrt{x}-2} \cdot \frac{(\sqrt{x}+2)}{\sqrt{x}+2}$$

$$(a-b)(a+b) = a^2-b^2$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \lim_{x\to 4}(\sqrt{x}+2) = \sqrt{4}+2 = \boxed{4}$$

Given $f(x) = 2x^2 + 3x + 1$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2+3x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+3x} + 3h \cancel{+1} \cancel{-2x^2} \cancel{-3x} \cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 + 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$= \lim_{h\to 0} (4x+2h+3) = 4x + 2\cdot 0 + 3 = \boxed{4x+3}$$

slope of tangent line to $f(x)$ @ any pt. $(x, f(x))$

The Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n\cdot(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{k(k-1)\cdots 3\cdot 2\cdot 1\,(n-k)!}$$

Pascal's $\triangle$

1		$(a+b)^0 = 1$
1 1		$(a+b)^1 = 1a + 1b$
1 2 1		$(a+b)^2 = 1a^2 + 2ab + 1b^2$
1 3 3 1		$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$
1 4 6 4 1		$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$$f(x) = x^3$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x^3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} (3x^2+3xh+h^2)$$

$$= 3x^2 + 3x(0) + 0^2 = \boxed{3x^2}$$