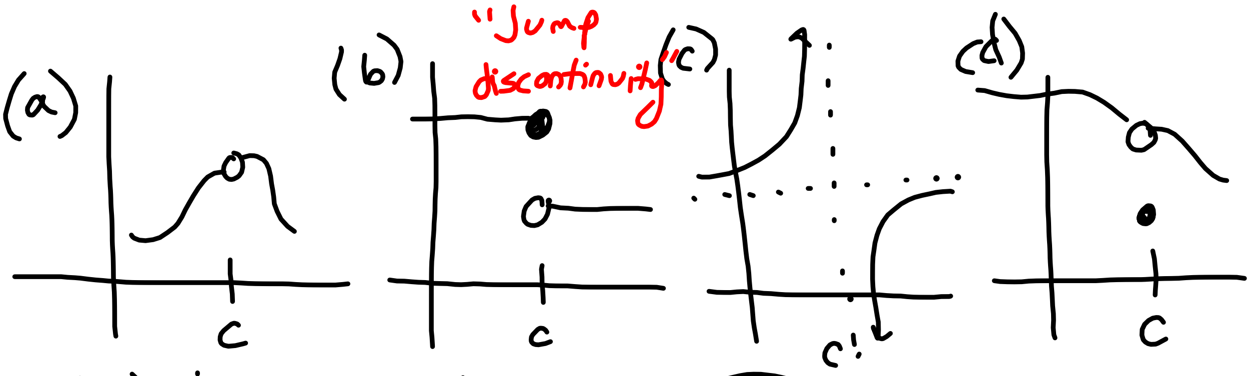
1.4 Continuity and One-Sided Limits

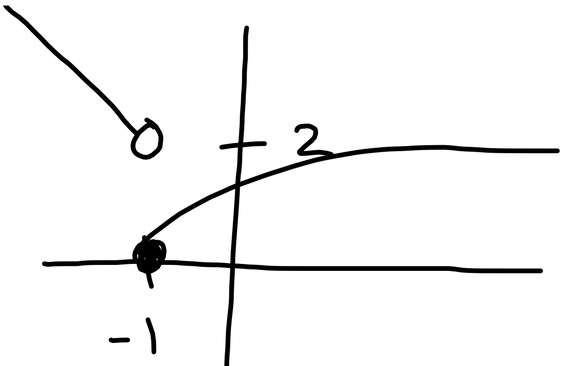


(a): f(c) is undefined

(b) and (c): $\lim_{x \to c} f(x)$ does not exist

(d): $\lim_{x \to c} f(x) \neq f(c)$

These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x \to -1^-} f(x) = 2$$

$$\lim_{x \to -1^+} f(x) = 0$$

$$\lim_{x \to -1} f(x) = \text{DNE}$$

One-Sided Limits

$\lim_{x \to c^+} f(x) = L$ *limit from the right*

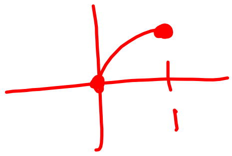
$\lim_{x \to c^-} f(x) = L$ *limit from the left*

$$\boxed{\lim_{x \to c} f(x) = L \ \textit{if and only if} \quad \lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)}$$

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$



$f(x)$ is continuous at c if

$$\lim_{x\to c} f(x) = f(c)$$

$[a,b]$

Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty,\infty)$ is everywhere continuous.

Continuity on a closed interval

A function f is continuous on the closed interval $[a,b]$ if it is continuous on the open interval I (a,b) and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$.

10. $$\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

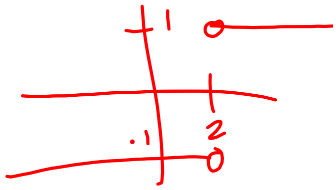
$$= \lim_{x\to 4^-} \frac{x-4}{(x-4)(\sqrt{x}+2)} = \frac{1}{4}$$

12. $\lim_{x \to 2^+} \frac{|x-2|}{x-2} = \boxed{1}$

$|f(x)| = f(x), \ f(x) > 0$

$|f(x)| = -f(x), \ f(x) < 0$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & x-2 > 0,\ x > 2 \\ \frac{-(x-2)}{x-2} = -1, & x-2 < 0,\ x < 2 \end{cases}$$



1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)}$$

removable discontinuity — hole @ $x=2$

non-removable discontinuity — vertical asymptote @ $x=-1$

f is continuous on: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

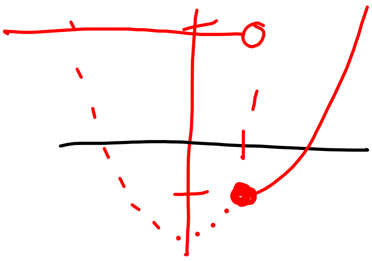
$$f(x) = \frac{|x-2|}{x-2}$$

non-removable jump discontinuity @ $x=2$

f is continuous on

$(-\infty, 2) \cup (2, \infty)$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

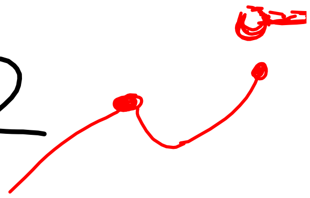


$1^2-2=-1$

$5=5$

non-removable jump discontinuity @ $x=1$

f is continuous on $(-\infty, 1) \cup [1, \infty)$



$-2+6=4$

$(-2)^2=4$, $3^2=9$

$$f(x)=\begin{cases} x+6, & x\le -2 \\ x^2, & -2<x\le 3 \\ 8 & x>3 \end{cases}$$

non-removable jump discontinuity @ $x=3$

f is continuous on $(-\infty,3]\cup(3,\infty)$