62. $\nearrow f(x)=\frac{1}{\sqrt{x}}$, $g(x)=x-1$ $\leftarrow$ continuous on its domain $(-\infty,\infty)$

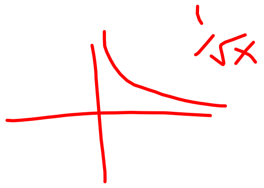
continuous on its domain $(0,\infty)$

Discuss the continuity of $f(g(x))$.

$(f\circ g)(x)=\frac{1}{\sqrt{x-1}}$

$x-1>0$

domain: $\{x \mid x>1\}$



continuous on $(1,\infty)$

64. $f(x)=\sin x$; $g(x)=x^2$

both f & g are continuous on $(-\infty,\infty)$

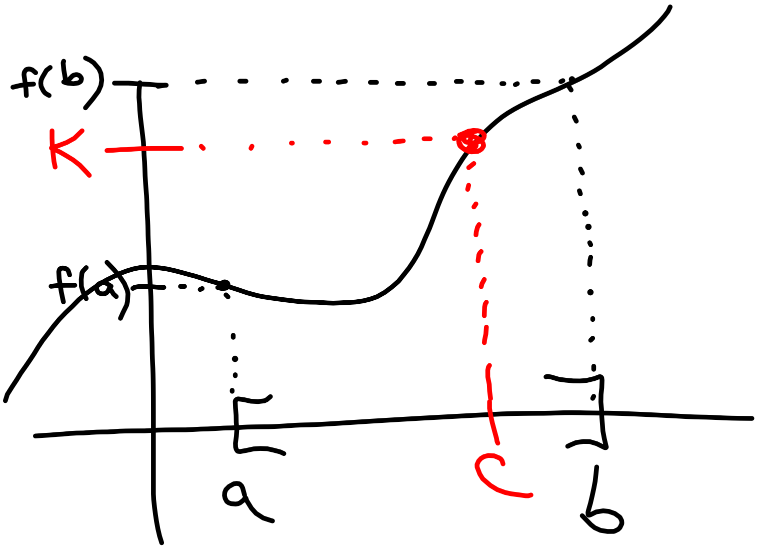
discuss the continuity of $f(g(x))$

$(f\circ g)(x)=\sin(x^2)$

continuous on $(-\infty,\infty)$

Intermediate Value Theorem

If *f* is continuous on the closed interval [*a*,*b*] and *k* is any number between *f(a)* and *f(b)*, then there is at least one number *c* in [*a*,*b*] such that *f(c)=k*.



Does the IVT guarantee a zero in the given interval?

76. $f(x)=x^3+3x-2$, $[0,1]$

$f(0) = -2 < 0$

$f(1) = 1+3-2 = 2 > 0$

IVT applies

$x^3+3x-2=0$

$x = $ ~

84. $f(x) = x^2 - 6x + 8$; $[0, 3]$ $f(c) = 0$

$$\left.\begin{array}{l} f(0) = 8 > 0 \\ f(3) = -1 < 0 \end{array}\right\} \text{IVT applies}$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$\boxed{x = 2}, \cancel{4} \notin [0, 3]$$

86. $f(x) = \frac{x^2 + x}{x - 1}$, $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$

$$f(5/2) = \frac{(5/2)^2 + 5/2}{5/2 - 1} = \frac{\frac{25}{4} + \frac{10}{4}}{5/2 - 2/2} = \frac{35}{4}\Big/_{3/2} = \frac{35}{\cancel{4}_2} \cdot \frac{\cancel{2}}{3} = \frac{35}{6} < 6$$

$$f(4) = \frac{4^2 + 4}{4 - 1} = \frac{20}{3} > 6$$

IVT

$$\frac{x^2 + x}{x - 1} = 6 \Rightarrow x^2 + x = 6(x - 1)$$

$$x^2 + x = 6x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2, \boxed{3}$$

1.5

Infinite Limits

$$\lim_{x\to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x\to c} f(x) = \infty$ *and* $\lim_{x\to c} g(x) = L$

1. $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

2. $\lim_{x\to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x\to c} \frac{g(x)}{f(x)} = 0$

$\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{1,000,000}, \ldots \rightarrow 0$

$-\frac{1}{10}, \frac{-1}{100}, \frac{-1}{1000}, \ldots \rightarrow 0$

$\frac{1}{10}, -\frac{1}{100}, \frac{1}{1000}, -\frac{1}{10,000}, \ldots \rightarrow 0$

Find the vertical asymptotes (if any).

14. $f(x) = \frac{-4x}{x^2+4}$

N/A

24. $h(x) = \frac{x^2-4}{\underbrace{x^3+2x^2}+\underbrace{x+2}} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x^2+1)}$

$x^2(x+2)+1(x+2)$

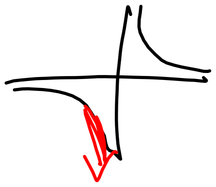
hole @ -2

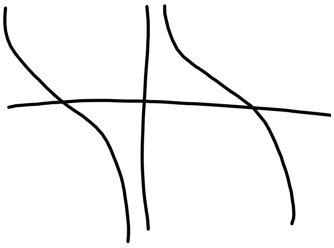
N/A

28. $g(\theta) = \frac{\tan\theta}{\theta}$

42. $\lim_{x\to 0^-}\left(x^2-\frac{1}{x}\right) = 0-(-\infty)$

$= \boxed{\infty}$





46. $\lim_{x\to 0}\frac{x+2}{\cot x} = \frac{2}{\pm\infty} \to \boxed{0}$

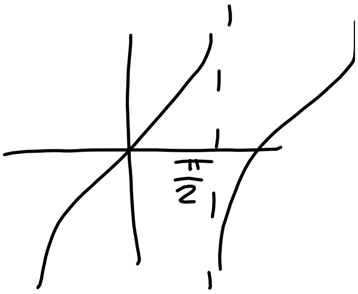
$= \lim_{x\to 0}(x+2)(\tan x)$

$= 2\cdot 0 = 0$

48. $\lim_{x\to\frac{1}{2}} x^2\tan\pi x$ DNE

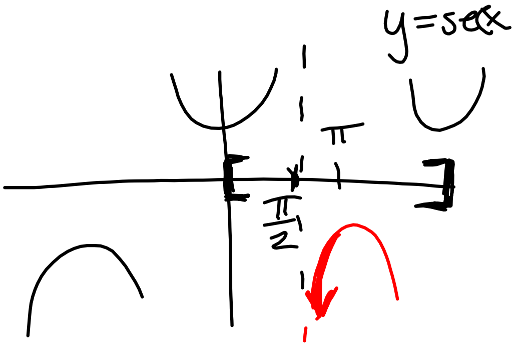
$= \left(\lim_{x\to\frac{1}{2}} x^2\right)\left(\lim_{x\to\frac{1}{2}}\tan\pi x\right)$

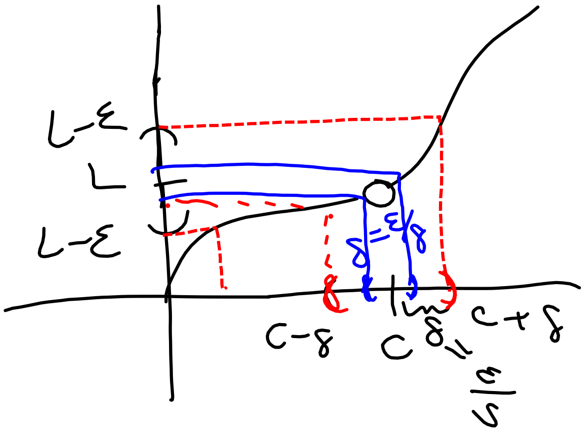
$= \frac{1}{4} \cdot$ $+\infty$ as $x\to\frac{1}{2}^-$; $-\infty$ as $x\to\frac{1}{2}^+$



$\lim_{x\to\frac{1}{2}^-} x^2\tan\pi x=\infty$; $\lim_{x\to 1/2^+} x^2\tan\pi x = -\infty$

52. $\lim_{x\to3^+}\sec\frac{\pi x}{6} = \boxed{-\infty}$





$f(x)=3x+2$

$\lim_{x\to1} f(x)=5$

$|f(x)-L|=|3x+2-5|$

$=|3x-3|=\frac{3|x-1|}{3}<\frac{\varepsilon}{3}$

$\delta=\varepsilon/3$

(a) ε (b) $\varepsilon/2$ (c) 3ε (d) $\varepsilon/6$ [✓ (e) $\varepsilon/4$]

Which is the largest δ that works?